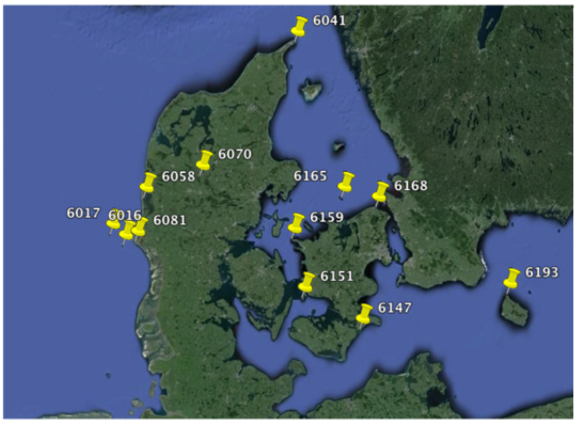
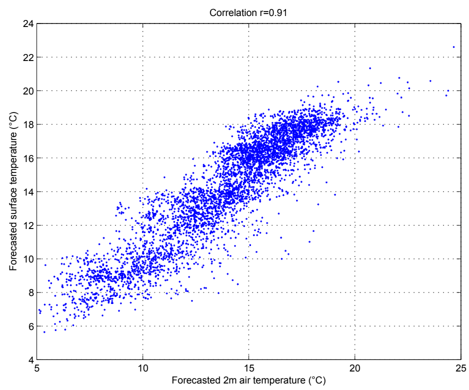
Dmi
Ministry of Climate and Energy

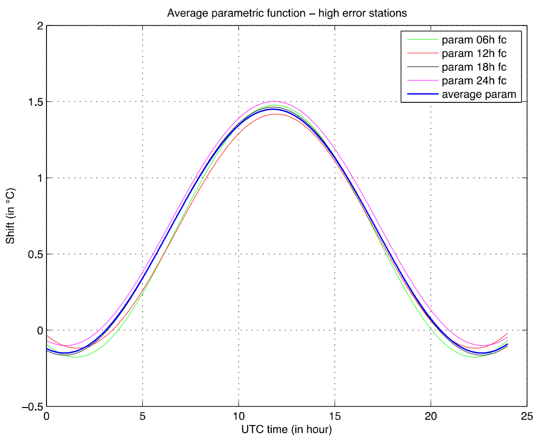
Scientific Report 13-03

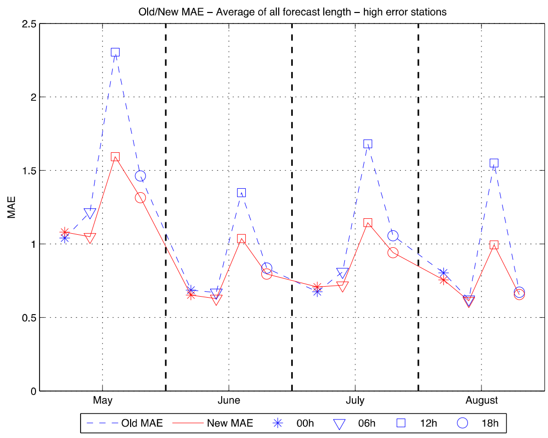
Analysis and Improvement of Statistical Error in Forecasted Air Temperature

Nicolas Gilet [1], Alexander Mahura [2], Claus Petersen [2], Bent Hansen Sass [2]









[1] *Institut National des Sciences Appliquées, INSA, 135 Avenue de Rangueil, 31400, Toulouse, France*
[2] *Danish Meteorological Institute, DMI, Lyngbyvej 100, DK-2100, Copenhagen, Denmark*

Copenhagen 2013

Colophon

Serial title:
Scientific Report **13-03**

Title:
Analysis and improvement of statistical error in forecasted air temperature

Subtitle:

Author(s):
Nicolas Gilet, Alexander Mahura, Claus Petersen, Bent Hansen Sass

Other contributors:

Responsible institution:
Danish Meteorological Institute

Language:
English

Keywords:
Correction – 2m air temperature – Time series – Parameterization – BIAS – MAE

Url:
www.dmi.dk/dmi/sr13-03.pdf

Digital ISBN:
978-87-7478-646-7 (on-line)

ISSN:
1399-1949 (on-line)

Version:
-

Website:
www.dmi.dk

Content:

Abstract

The main aim of this study is to improve forecasted 2 meter air temperature from the numerical weather prediction (NWP) model (called – HIRLAM – High Resolution Limited Area Model) output. The specific objectives are: 1) study behavior in the errors (biases) for selected Danish synoptical stations, where 2m air temperature is measured; and 2) propose, based on statistical approach, a parameterization to correct such errors.
A dataset of forecasts and observations (long-term time-series: 1 May - 31 August 2012) for sixty six stations had been analyzed. It has been found that the involved meteorological parameters followed several criteria (distribution close to normal, periodicity on a diurnal cycle and strong correlation) which are necessary to have an efficient correction of the air temperature. The developed parameterization is based on a shift of the error density function for 2m air temperature compared with the standard normal distribution shape. The parameterization describing correction to temperature is based on a trigonometric function and can be applied for any forecast length.

Resume

Formålet med dette projekt er at forbedre prognosekvaliteten af det rå output af 2 m temperaturen fra den numeriske vejrmodel(HIRLAM-HIgh Resolution Limited Area Model). Følgende emner er blevet studeret: 1)Undersøgelse af fejl(bias) for Danske synoptiske stationer, hvor 2 m temperaturen males, 2)Udarbejde en statistisk metode til at reducere fejlene.
Et dataset bestående af prognoser og tilhørende observationer dækkende perioden 1 maj -31 august 2012 for 66 stationer blev udarbjdet og anvendt i analysen. Analysen viste at de anvendte meteorologiske parametre opfylde flere nødvendige kreterier(næsten normalt fordelt, periodisk på daglig basis samt stærk korrelation), hvilket er nødvendigt for en effektiv metode til temperatur korrektionen. Den udviklede metode foretager et skift i fejlfunktionen for 2 m temperatur sammenlignet med en standard normalfordeling. Metoden til at korrigere temperaturen er baseret på trigometriske funktioner og kan anvendes for en hver prognose længde.

Keywords: Correction – 2m air temperature – Time series – Parameterization – BIAS – MAE

1. Introduction

The 2 meter air temperature (T2m) is one of the most important output parameters from the numerical weather prediction (NWP) modeling. This parameter is widely used in many Danish Meteorological Institute's (DMI) applications such as DMI's "city weather" which is graphically available as a time-series from DMI's webpage. Here, the end-user (or customer) of the webpage can get the weather at exact locations (in particular, the cities) several days ahead, and where DMI's local NWP model output is used as input for the first 48 hours of forecast.
This parameter is also a part of the standard verification of the performance of the NWP model and it is an important competition parameter with other providers of the weather forecasts. For these reasons it is important to get the optimal performance out of the NWP model. Even though the model is under constant development and improvement, it is known from verifications that the modeled 2m air temperature has always systematical errors under certain meteorological conditions. The model is verified against observations by interpolating the forecasted 2m air temperature to the observational points. In general, the difference between the observed and the forecasted temperatures is a consequence of different physical processes. There are several relevant errors such as:

- Interpolation error ,
- Observation error ,
- Model errors ,
- Initial error (Analysis error) ,
- Numerical errors , and
- Unknown error.

The non-linear Navier-Stocks equations, whose are solved in NWP, will in general lead to an increasing error in the model state as a function of the forecast length (*Petterssen, 1969*; *Wallace and Hobbs, 1977*). This error will most likely not be systematic, and it would not be possible to correct. However, other aspects and, in particular, shortcomings in the model will result in errors which might be systematically depending on the weather type, time of the day and forecast length.
This study has focus on 2m air temperature as it has been noticed that the HIRLAM operational model (SKA, 3 km resolution) have systematically errors in the 2m temperature which tend to be too low in a day time. It is assumed that the error is a result of inaccuracies in the NWP model description of the surface temperature which leads to increased evaporation. This report focuses on model statistics related to the 2m air temperature and how the error is related to other output parameters of the model. Finally, the statistical approach to correction of the air temperature for selected synoptical stations will be elaborated, tested and verified for selected periods of time.

This research is a part of the DMI's project related to improvement of the air temperature forecasts for the "city weather" applications produced by operational HIRLAM model (see more details in *Mahura et al., 2013*) with a focus on improvement of stations having the largest biases or errors in forecasts.

Details of research work done in this study were summarized and presented during weekly seminars (in total 8). In this report, the most interesting and useful findings (with corresponding illustration material) are summarized. A summary of the completed tasks (linked with corresponding presentations) includes the following:

1. Introduction into the basic statistics; calculation of BIAS and mean absolute error (MAE) for selected synoptical station (WMO-6058) for 6h forecast length (FL) for each period of the day (e.g. 00, 06, 12 and 18 UTCs); analyses of distributions, correlations and periodicities for selected meteorological parameters (2m air temperature – T2m, dew temperature – Td2m, surface temperature - Ts, and wind speed - U);

2. Estimation of correlations of meteorological parameters (T2m, Td2m, U) errors for each month and day in May 2012; relation between cloud cover for night- vs. day-time; selection of high error stations (HES) group; calculation of BIASes and MAEs; estimation of distribution density of all meteorological parameters and T2m errors (at 00, 06, 12, and 18 UTCs) for HES for each month (May, Jun, Jul, and Aug 2012) and comparison of correlations between a selected HES and random stations.
3. Comparison of BIAS and MAE for each time of day for every FL for HES; analysis of estimated distribution density of T2m error for every FL compared with the time of day; testing Lagrange interpolation for data fitting of the shift in T2m error for selected station (WMO-6016) for each FL and month; evolution of correlation from May to August 2012 for each time of day and for each FL between T2m and Td2m.
4. Calculation of averaged polynomial functions for the HES group on a diurnal cycle for each month and FL; analysis of improvements (new BIASes and MAEs); interpolation by cubic spline for shift in T2m error for 4 months period (from May till August 2012), calculation of averaged polynomial function and improvement.
5. Automatization procedure to calculate averaged polynomial function on a diurnal cycle for group of stations randomly selected; classification of 66 Danish meteorological stations based on geographical location; calculation and comparison of BIASes and MAEs, averaged polynomial function and improvement for each classified group of stations.
6. Development of parameterization with averaged polynomial fit to data for each month for HES and airport station (AS) groups; estimation of improvement (for percentage, and value for BIAS and MAE).
7. Evaluation of T2m error with a gradation (small, medium and large errors), histogram analysis for wind speed, heat flux, and hours of day and cloud cover on a diurnal cycle; estimation correlation between heat flux and T2m error for selected inland vs. HES station.
8. Evaluation of T2m error compared with error of the cloud cover; correlation between heat flux, cloud cover and T2m; correlation between T2m error and heat flux in day- vs. night-time for each group of gradation error.

2. Methodology

2.1. Danish meteorological stations

In the dataset used in this report, sixty-six Danish synoptical stations having a long-term time-series of meteorological observations were selected. However, only two groups of the Danish meteorological stations are considered here in details: 1) group of stations called "high error stations" (HES), which includes stations where the mean absolute error, MAE (see section 2.3) is higher than 1°C after a first correction (*Mahura et al, 2013*) and 2) airport stations (AS).

Table 1*: Summary of classification of the Danish synoptical stations.*

High error stations (HES)	Airport Stations (AS)	Classification based on geographical position				
		Sea	Urban		Rural	
			Inland	Coastal	Inland	Coastal
6016	6030	6016	6074	6033	6031	6019
6017	6060	6017	6102	6051	6032	6041
6041	6070	6183	6109	6052	6049	6073
6058	6080		6126	6058	6056	6063
6147	6104		6188	6088	6065	6079
6151	6110		6141	6123	6068	6081
6159	6118		6160	6132	6069	6093
6165	6120			6136	6072	6119
6168	6170			6138	6082	6096
6193	6180			6149	6116	6147
	6108			6168	6135	6151
	6124			6169	6154	6159
	6190			6174	6156	6165
				6181		6179
				6197		6193
10	13	3	7	15	13	15

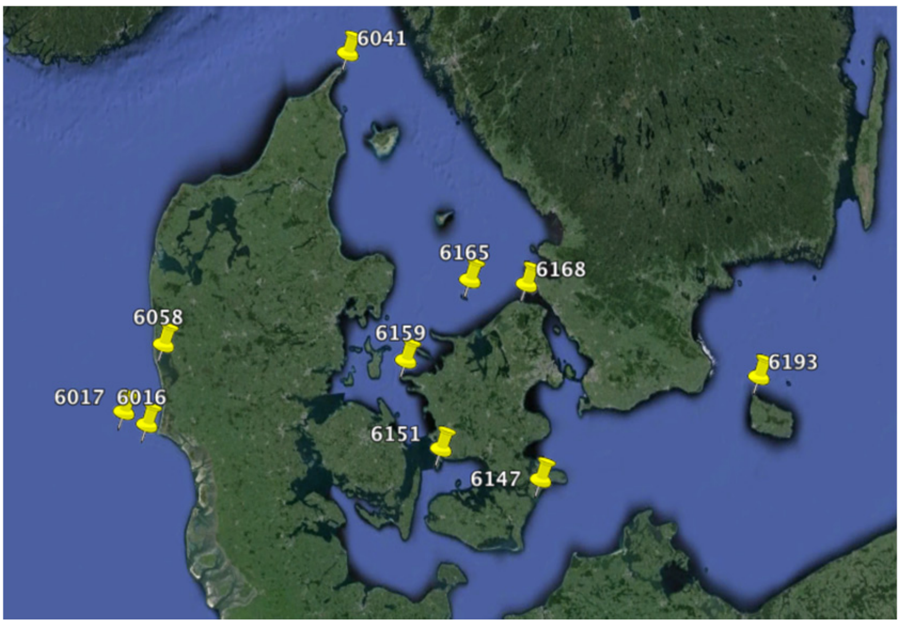


Figure 1*: Geographical positions (extracted from Google-Earth) of the high error stations.*

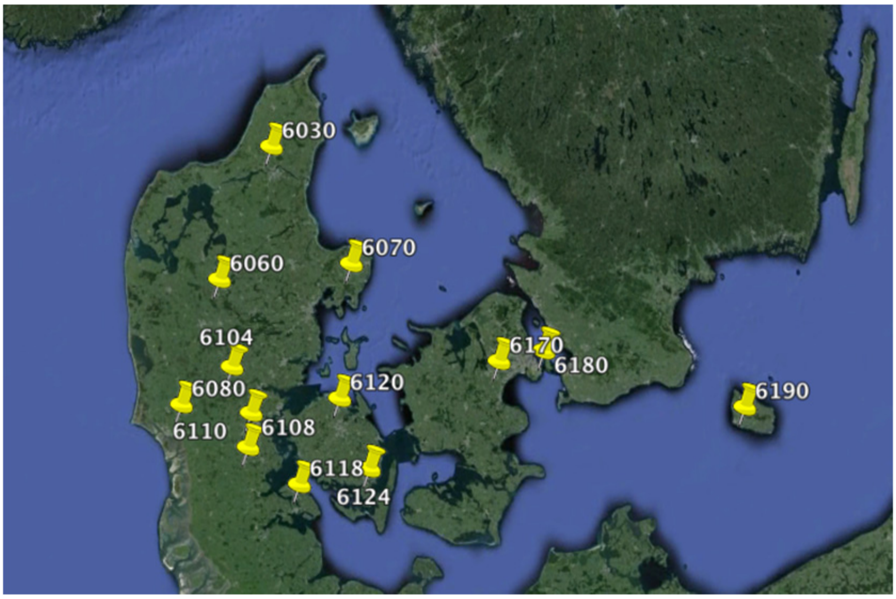


***Figure 2**: Geographical positions (extracted from Google-Earth) of the airport stations.*

All of these 66 stations are classified in Table 1. The first two columns iterate the ID of stations on the groups. The after-following columns classify all of stations compared with their geographical locations: sea, in urban area near the coast or inland, and in rural area near the coast or inland. Note the airport stations are not classified as urban or rural related stations. Geographical positions of the first two groups of stations are presented in Figures 1 and 2. Figure 1 shows the group of the high error stations which, as it seen, are all situated only in the coastal area.

2.2. Time series of observations and forecasts

The dataset of 66 Danish stations contains the forecasted and observed values of 4 meteorological parameters: wind speed (m/s), 2m air temperature (°C), dew point temperature (°C) and surface temperature (°C). For the last parameter, the observed values were missing. The dataset includes four observations daily (at 00, 06, 12 and 18h UTC+00) from 1st May till 31 August 2012. Originally, in total 8 observations are performed each day (every 3h from midnight in UTC time) at each synoptical station, but the forecasted values – used - are written out only at every 6 hour (00, 06, 12, and 18 in UTC time) interval. So, to compare forecasted and observed values, only measurement data on times of forecasts are taken. Summary on available measurements used in our analysis is given in the Appendix C.
The NWP forecasts used in this study were made for 4 forecast lengths (06h, 12h, 18h and 24h). For each station, there are 123 days with 4 corresponding observations for each day. So, in total, for each forecast length there are 492 (123 x 4) records for each meteorological parameter.
Note that some basics on statistical analysis of meteorological parameters are described by *Hartmann (2013)* and were applied for this study.

2.3. Indicators for analysis

For this study, two indicators are widely used: the BIAS (mean error) and the MAE (Mean Absolute Error).
The formulae of the BIAS and MAE are defined as the following:

$$BIAS = \frac{1}{n}\sum(f_i - o_i) \quad (1)$$

and

$$MAE = \frac{1}{n}\sum | f_i - o_i | \quad (2)$$

where n is the number of observations, and f and o are respectively the forecasted and observed values of meteorological parameters.
BIAS indicates if the model is too 'cold' (negative BIAS) or too 'hot' (positive BIAS) whereas MAE is the average absolute error. The perfect forecast has 0 BIAS and 0 MAE, which is very rare case for the numerical model. A large BIAS is an indication of systematical errors whereas large MAE is an indication of a 'bad' quality forecast.
These indicators will be used to identify which stations have the largest potential for correction of temperature errors. The high error stations in HES group were selected as stations having a MAE higher than 1°C after a first correction.

2.4. Normal distribution, periodicity and correlation for meteorological parameters

There might be physical reasons for errors in meteorological parameters, in particular in air temperature, which can be related to daily variation, such as solar radiation, sea breeze circulation, etc. It has been examined if the used forecasts and observations have followed three main criteria or properties in order to construct an effective correction method:

- Parameters have distribution close to a normal distribution. A normal density allows correcting more easily the error. If x follows a normal law, i.e. $X \sim \mathcal{N}(\mu, \sigma)$, the density function is given by :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad (3)$$

  where: μ is the mean and σ is the standard deviation.
  Figure 3 shows two general examples of normal distribution. The first (blue) curve has the standard normal distribution ($\mu = 0$ and$\sigma = 1$). The second (red) curve describes the normal distribution with a mean equal to 1 and a standard deviation equal to 2, and with a slight displacement to the right from the 0.

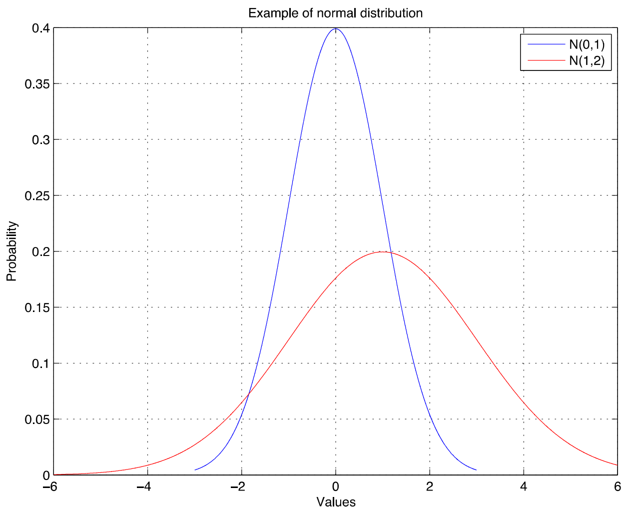


Figure 3: *General examples of normal distributions*

- Parameters should show variability on a diurnal cycle or periodicity. Such periodicity can show existence of potential correction on a day-by-day scale.

- Parameters should show a strong correlation as well. Strong correlations are defined here as: higher than +0.4 or smaller than -0.4. A strong correlation allows correcting at the same time the other parameters as well. If $X = (x_i)_{i=1:n}$ and $Y = (y_i)_{i=1:n}$ are a vectors of data with the same length, correlation is given by:

$$Cor = \frac{Cov(X,Y)}{std(X).std(Y)} = \frac{\sum(x_i - \bar{x}).(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})}\sqrt{\sum(y_i - \bar{y})}} \quad (4)$$

where $\bar{x}$ and $\bar{y}$ are respectively the mean of X and Y.
Values of correlation are situated between -1 and +1. The perfect correlation is obtained if it is equal to +1 or -1.

2.5. Interpolation for data fitting

For analysis of the T2m error shifts (see section 3.2), a method to connect a finite number of points with a function is necessary. Such method is called interpolation. The simplest method is to draw a sequence of straight lines connecting these points. However, a more advanced method, for example, with a polynomial function is more efficient to fit meteorological parameters (*Hartmann, 2013).* In this report, the method of the Lagrange interpolation is used to link the points on a diurnal cycle. Explanation of this method is given in this section.

The connecting function is a fourth-order polynomial i.e. $P(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ where a_i is the unknown values. Values (written y_i) of this polynomial function are known on connected points, i.e. $P(x_i) = y_i$. In our case, there are 5 points to connect (see section 3.2). So, a system of 5 equations can be written as:

$$\begin{cases} a_4x_0^4 + a_3x_0^3 + a_2x_0^2 + a_1x_0 + a_0 = y_0 \\ a_4x_1^4 + a_3x_1^3 + a_2x_1^2 + a_1x_1 + a_0 = y_1 \\ a_4x_2^4 + a_3x_2^3 + a_2x_2^2 + a_1x_2 + a_0 = y_2 \\ a_4x_3^4 + a_3x_3^3 + a_2x_3^2 + a_1x_3 + a_0 = y_3 \\ a_4x_4^4 + a_3x_4^3 + a_2x_4^2 + a_1x_4 + a_0 = y_4 \end{cases} \quad (5)$$

Then, this problem can be solved by constructing a linear system following this form:

$$\begin{bmatrix} x_0^4 & x_0^3 & x_0^2 & x_0 & 1 \\ x_1^4 & x_1^3 & x_1^2 & x_1 & 1 \\ x_2^4 & x_2^3 & x_2^2 & x_2 & 1 \\ x_3^4 & x_3^3 & x_3^2 & x_3 & 1 \\ x_4^4 & x_4^3 & x_4^2 & x_4 & 1 \end{bmatrix} \begin{bmatrix} a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \quad (6)$$

with $x_0 = 0,\ x_1 = 6,\ x_2 = 12,\ x_3 = 18\ and\ x_4 = 24$. (UTC time)

Before trying to find solution, it is critically important to calculate the conditionnement of the constructed matrix (see left-hand side of (6)). In fact, a small value of conditionnement brings a

small dependence of the problem (6) compared with the original data. In other words, a small change of data values should not change completely the result of the solution of the stated problem. Thus, if the conditionnement is large (e.g. higher than 10^7), this is also not efficient to resolve the stated problem. So, in our case, the calculated matrix conditionnement is equal to 4.10^5 which shows a good sign for solution of the stated problem.
Note that for an invertible matrix called A, the conditionnement is given by :

$$\kappa(A) = \| A^{-1} \| \| A \|$$

where $\| . \|$ is a matrix norm and A^{-1} is the inverse matrix of A.

2.6. Parameterization for improvement of 2m air temperature

A parameterization of the T2m error shift (see Figure 8b) for each forecast length for each month has been developed for the HES and AS groups of stations.
Note, the details on operational statistical correction procedure for air temperature forecasts are shown in *Mahura et al. (2013*) and are not discussed here, but results of this study – e.g. the newly developed parameterization - is the integral part for additional improvement of the overall performance of the correction procedure. For start, as an example, in this study the approach used by *Mahura et al. (2009)* was selected and further developed.

At the beginning, the following standard formulation of equation can be proposed:

$$S(t) = S(t_0) + \Delta S(t) \quad (7)$$

where :

$S(t)$ is the shift of 2m air temperature (T2m) error density peak at time *t* on a diurnal cycle,
$S(t_0)$ is the shift of T2m error at the beginning of the diurnal cycle (00h UTC time),
$\Delta S(t)$ is the change of the shift of T2m error due to the diurnal cycle, and
t is the time (hour).

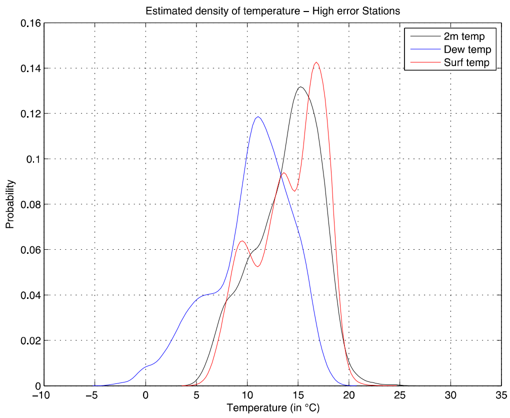
3. Results and Discussions

3.1. Analysis of criteria for meteorological parameters

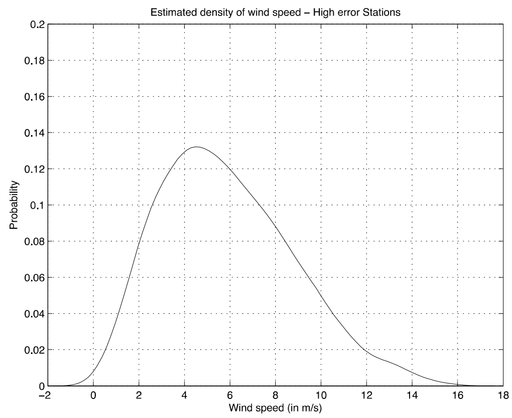
In section 2.4, it was stated that some properties must satisfy several criteria to make an efficient correction of 2m air temperature error. Results of this analysis are shown below.
Figure 4 shows the estimated distribution density of four meteorological parameters: wind speed (Figures 4b and d), and Figures 4a and 4b for 2m air temperature, dew temperature and surface temperature for the high error and airport stations. Ignoring 2.5% at each side of the distribution curves, as seen these parameters follow approximately a distribution close to normal. The same result was also obtained for other different forecast lengths of 12, 18, and 24 hours (not shown here).

Results show as expected (Figures 5b and 6b) a strong correlation (+0.9) between forecasted surface temperature and 2m air temperature for the two groups of stations. There are also large correlations (Figures 5c and 6c) between forecasted 2m air temperature and dew temperature for both groups of stations. However, there are low correlations (Figures 5a and 6a) between forecasted wind speed and 2m air temperature, and therefore, with forecasted surface and dew temperature (not shown). That means that the same type of correction of 2m air temperature cannot be applied for the wind

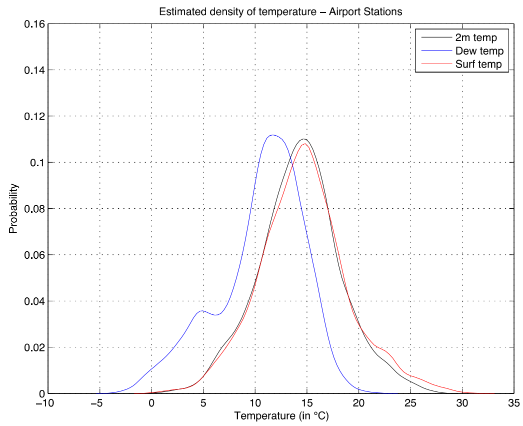
speed, but this correction also can be applied for the dew temperature and the surface temperature errors.
Correlations values were also approximately the same for the others forecast lengths (12h, 18h and 24h) for both groups HES and AS.



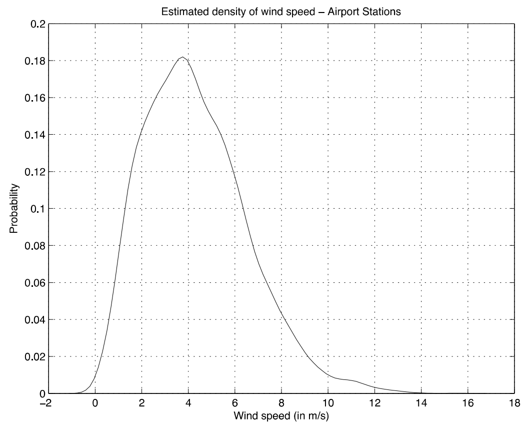
(a)



(b)

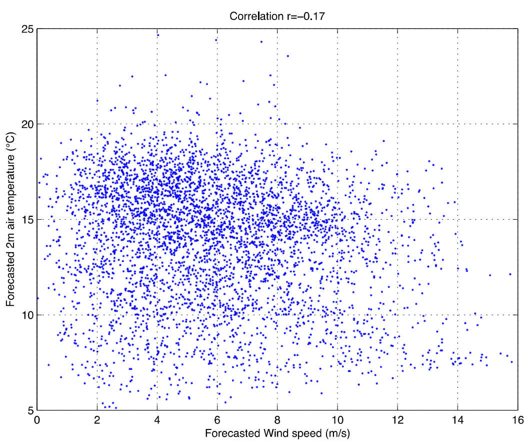


(c)

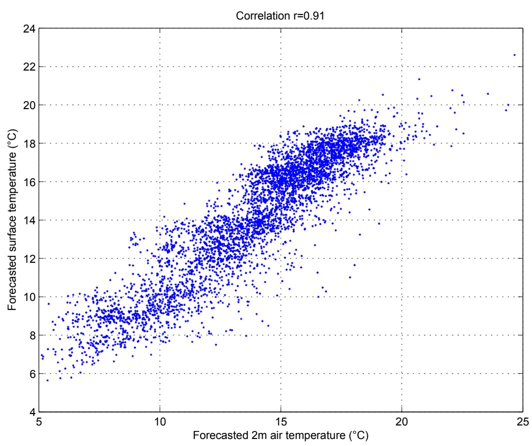


(d)

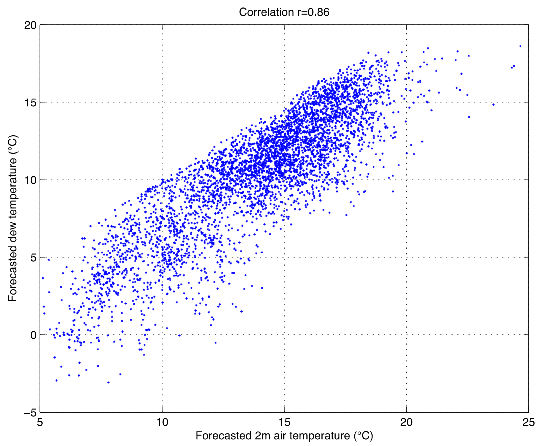
***Figure 4**: Estimated distribution density function of meteorological parameters: wind speed (c and d) and temperatures (a and b) for (a and b) HES group and (c and d) AS group for 6h forecast length.*



(a)

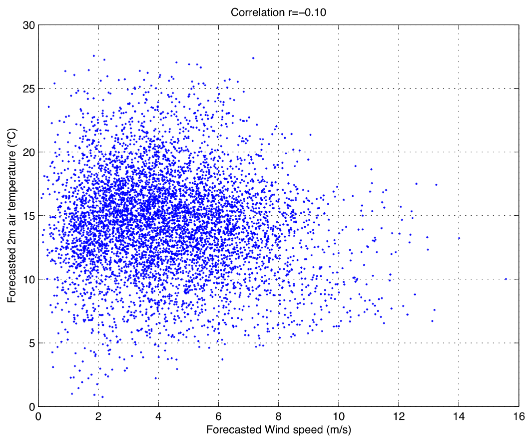


(b)

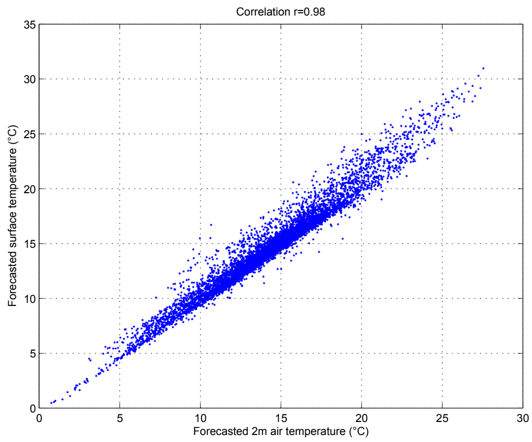


(c)

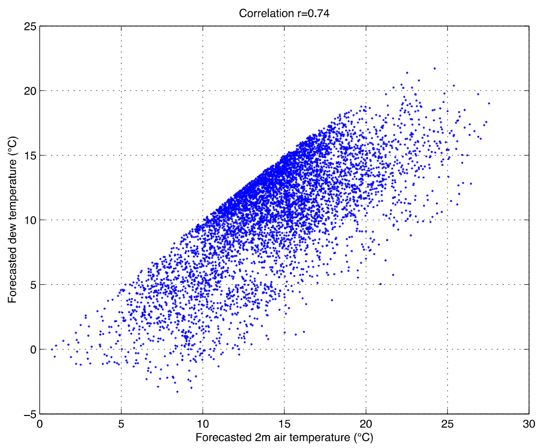
Figure 5: *Correlation between parameters for all data for 6h forecast length – **HES group***
(a) Correlation between wind speed and 2m air temperature
(b) Correlation between 2m air temperature and surface temperature
(c) Correlation between 2m air temperature and dew temperature



(a)



(b)



(c)

Figure 6: *Correlation between parameters for all data for 6h forecast length – **AS group***
(a) Correlation between wind speed and 2m air temperature
(b) Correlation between 2m air temperature and surface temperature
(c) Correlation between 2m air temperature and dew temperature

For the periodicity of meteorological parameters or variability on a diurnal cycle, several examples are presented on Figure 7 for the HES station (WMO-06058). A diurnal cycle is well observed for all temperatures (Figures 7a – T2m, 7b – Ts, and 7c – Td2m). The peeks of all temperatures are observed on mid-day and the colder temperatures are observed in night-time. Figure 8 shows that an amplitude of the air temperature is the largest especially during spring (May) day compared to summer (Jun-Aug) day. Similar results on periodicity were observed for the group of airport stations as well (not shown).

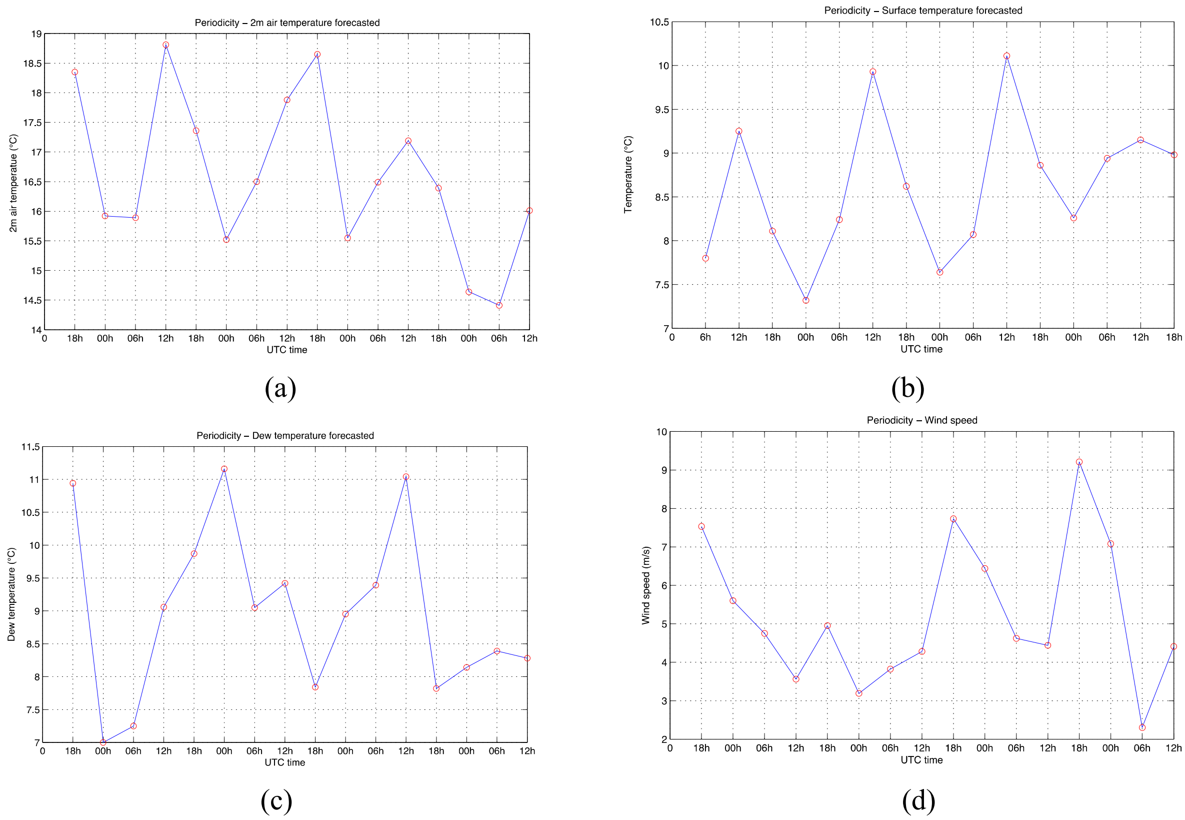
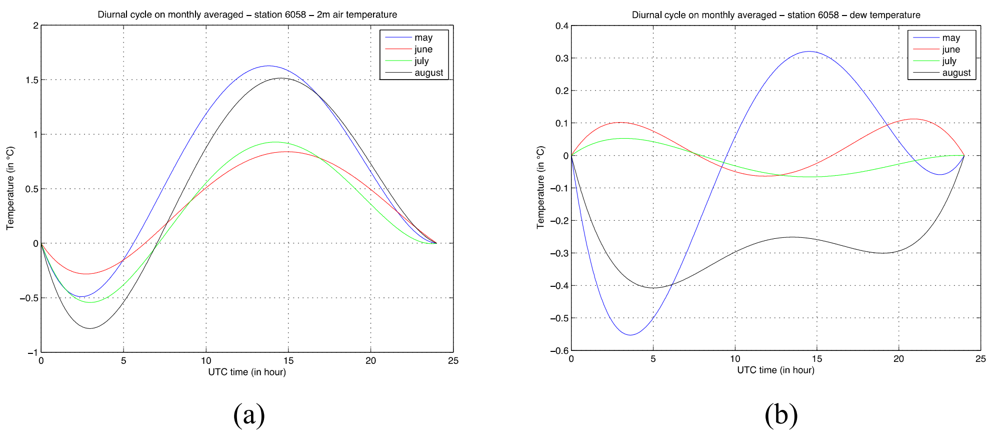
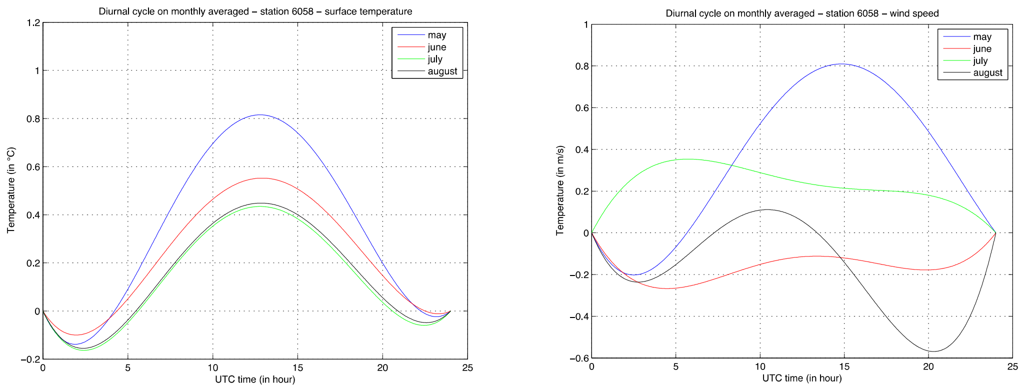


Figure 7: *Observed periodicity of (a) 2m air temperature, (b) surface temperature, (c) dew temperature and (d) wind speed during 3 days (randomly selected) for the HES station (WMO-06058).*



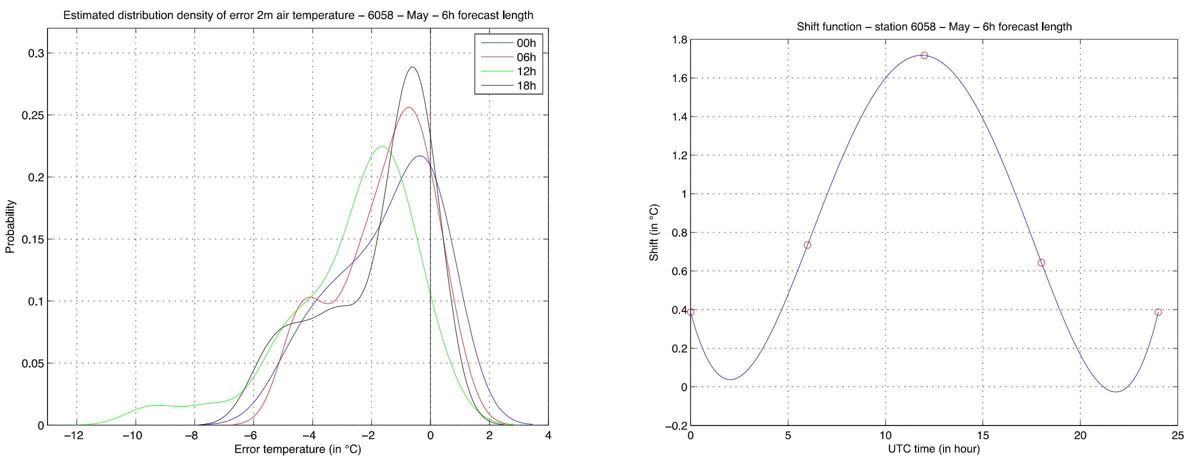


(c) (d)

***Figure 8**: Monthly averaged amplitude variability on a diurnal cycle (as difference between value of meteorological parameter minus value at 00h UTC time) for: 2m air temperature (a), dew temperature (b), surface temperature (c), wind speed (d) - for HES station (WMO-06058).*

3.2. Error of 2m air temperature on a diurnal cycle

In this part, the behavior of the error of 2m air temperature on a diurnal cycle monthly averaged will be shown. Figure 9a shows the estimated density of this T2m error on an example of the high error station (WMO-06058) for different parts of the day for May 2012 for 6 hour forecast length. All of these densities are shifted to the left compared with the origin (value 0 – see Figure 9a) which indicates an underestimation of the 2m air temperature. Calculated BIAS for this station shows negative values (see Table 2). Moreover, the BIAS is the highest at mid-day. The peak of estimated density curve for this period of the day is more shifted to the left than the other density curves (for other times).



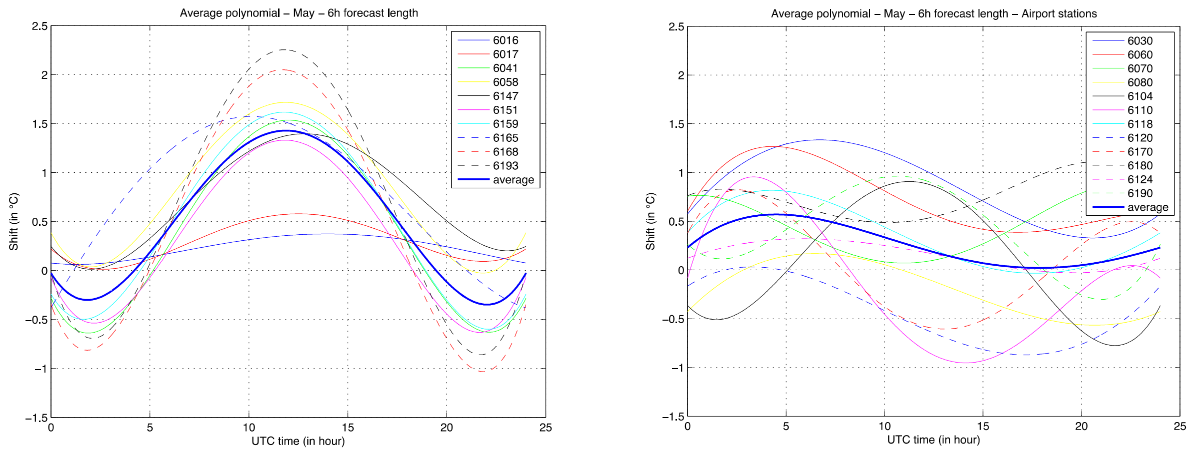
(a) *(b)*

***Figure 9**: (a) Estimated distribution density of the error of 2m temperature for the HES station (WMO-06058) for 6h forecast length (based on May 2012 data with different periods of day);*
(b) Shift of the estimated distribution density peak for the HES station (WMO-06058) for 6h forecast length (based on May 2012 data with different periods of day).

The T2m error shifts are calculated on a monthly averaged basis for each time of the day (00h, 06h, 12h and 18h in UTC time). These values are plotted on Figure 9b, and the Lagrange interpolation was applied to connect the points. The final averaged curve describes the behavior of the T2m error shift on a diurnal cycle on a monthly averaged basis for one HES station (WMO-06058). The curve which links the points is based on a fourth-order polynomial fit. The same analysis was realized for each of HES and AS stations (10 and 12 stations, respectively). Then, all curves were superimposed on one graph (Figures 10a and 10b). For each group of stations, a monthly averaged polynomial function was calculated for each of 4 forecast lengths. Results by months and forecast lengths are summarized in the Appendix A. Examples are also given on Figure 10 for the group of high error stations (see Figure 10a) and the airport stations (see Figure 10b).

***Table 2**: BIAS for the HES station (WMO-06058) for 6h forecast length (based on May-Aug 2012 data with different periods of day).*

Month	Period of the day, h			
	00	06	12	18
	Biases in deg C			
May	*-1.49*	*-1.60*	*-2.66*	*-1.80*
Jun	*-0.25*	*-0.62*	*-1.56*	*-1.06*
Jul	*-0.58*	*-0.83*	*-1.75*	*-1.18*
Aug	*-0.54*	*-0.56*	*-1.45*	*-0.64*



(a) *(b)*

***Figure 10**: Fitting polynomial function for all station in (a) HES and (b) AS groups and average polynomial fit for May 2012 for 6h forecast length.*

This figure shows that these groups have different behaviors. We can also see in Appendix A that the average polynomial functions for high error stations have almost the same behavior for every month (e.g. May, June, July and August 2012) and for each forecast length (06h, 12h, 18h and 24h). On contrary, the behavior of the airport stations isn't the same for different months and forecast lengths.

Finally, we have found that the averaged polynomials have a shape which can be parameterized by one of the standard trigonometric functions for both the HES and AS groups of stations for each forecast length and month.

3.3. Parameterization of the error of 2m air temperature

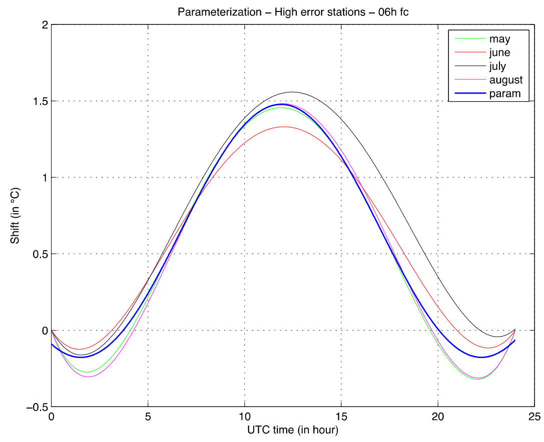
3.3.1. High error stations

For the group of the high error station, the following parameterization has been proposed:

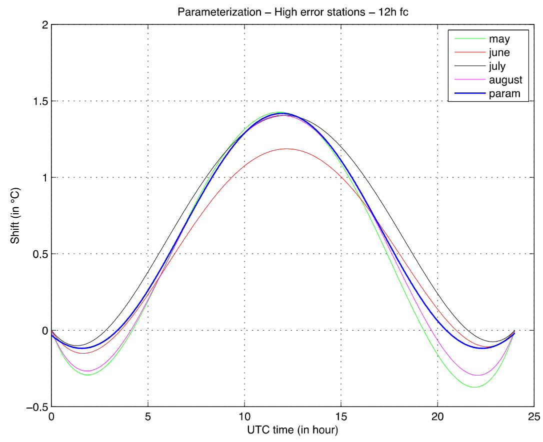
$$\Delta S(t) = A \cos\left(\pi + \frac{t - B}{C\pi}\right) + D \quad (8)$$

where A, B, C and D are empirical coefficients (B and C are dimensionless, and A and D have units

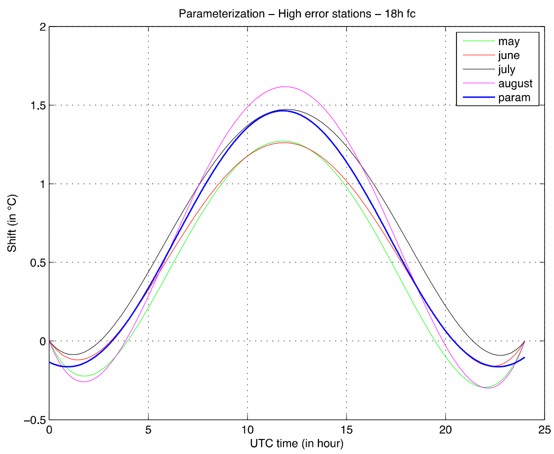
of convergence to temperature). The summary of the best fit is tabularized in Table 3.
Figure 11(a-d) shows that the parametric functions for each forecasted length are almost the same. Additionally, an averaged parametric function has been calculated and plotted on Figure 11e.



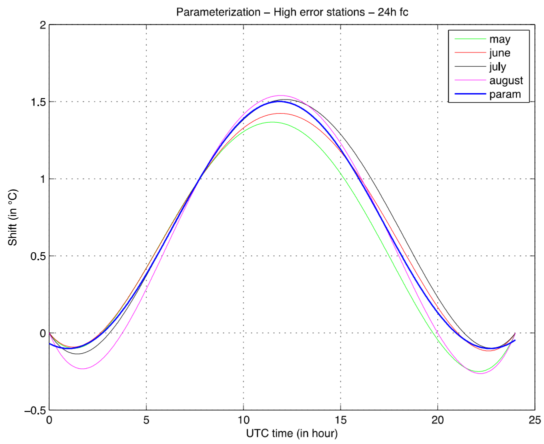
(a)



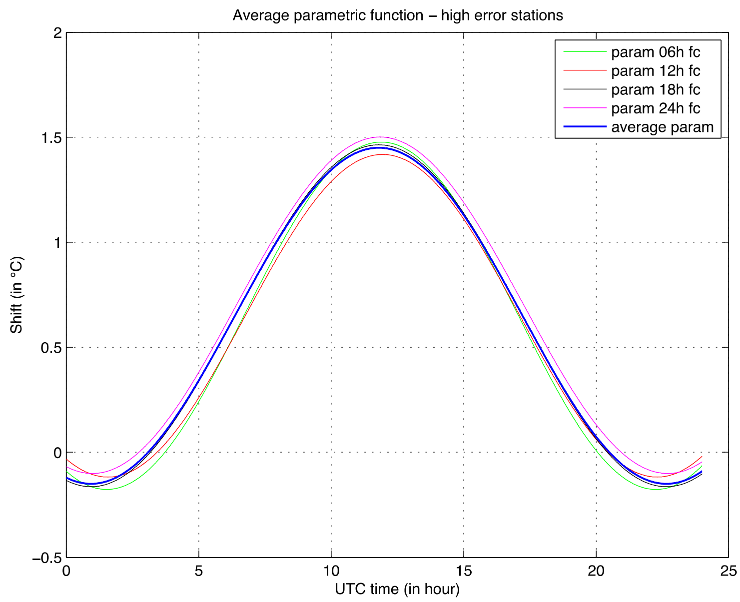
(b)



(c)



(d)



(e)

Figure 11*: Parameterization of T2m error shift for each forecast length: (a) 6h, (b) 12h (c) 18h, and (d) 24h; and (e) Averaged parametric functions for different forecast lengths and overall (6-12-18-24h) averaged - for the HES group of stations.*

Table 3: *Values of empirical coefficients in the parameterization formula for the best fit for the high error stations for May-Aug 2012.*

Reference to Figure	Forecast length	Empirical coefficients			
		A	B	C	D
11a	06h	0.83	22.25	1.05	0.65
11b	12h	0.77	22.30	1.05	0.65
11c	18h	0.81	22.65	1.10	0.65
11d	24h	0.80	22.70	1.10	0.70
11e	***Overall averaged***	***0.80***	***22.65***	***1.10***	***0.65***

Table 3 shows that the coefficients are almost the same between different forecast lengths. An interval was also calculated for these coefficients: A is changing within 0.80 $\pm$ 0.05, for B - 22.50 $\pm$ 0.20, for C - 1.05 $\pm$ 0.05, and for D - 0.65 $\pm$ 0.05.

3.3.2. Airport stations

For the airport stations (AS), the same parameterization as developed for the HES stations (see section 3.3.1) has been applied for the same months. It showed good results for May and Jul 2012. However, this parameterization was not efficient for Jun and Aug 2012. Results are presented on Figures 12 and 13. The best fit is tabularized in Table 4 (for May 2012) and in Table 5 (for Jul 2012). An overall averaged parametric function is shown on Figures 12e and 13e.

Table 4: *Values of empirical coefficients in the parameterization formula for the best fit for the airport stations for May 2012.*

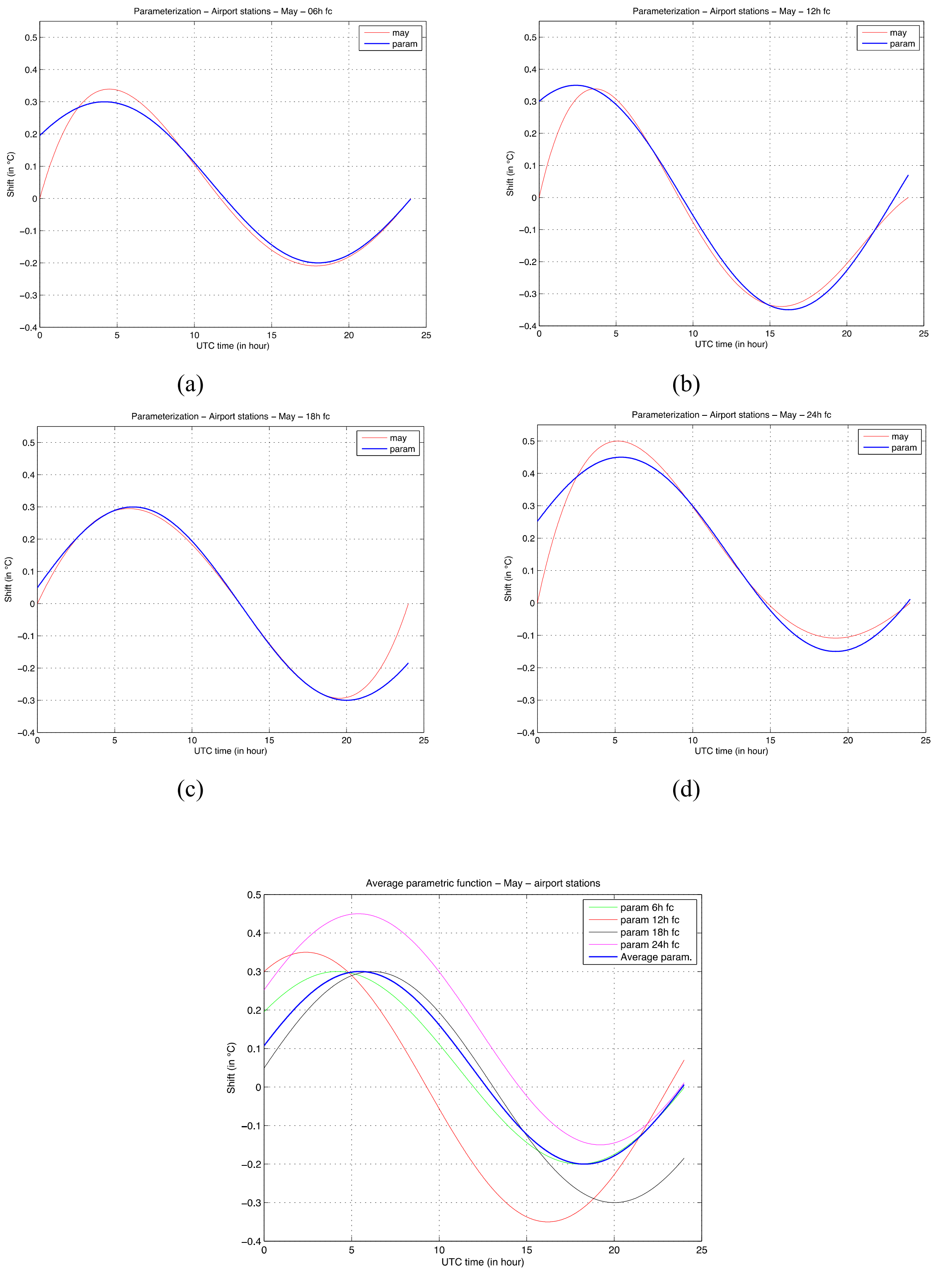
Reference to Figure	Forecast length	Empirical coefficients			
		A	B	C	D
12a	06h	0.25	18.0	1.4	0.05
12b	12h	0.35	16.2	1.4	0.00
12c	18h	0.30	20.0	1.4	0.00
12d	24h	0.30	19.2	1.4	0.15
12e	***Overall averaged***	***0.25***	***18.3***	***1.3***	***0.05***

An interval was also calculated for these coefficients: A is changing 0.30 $\pm$ 0.05, for B – 18.0 $\pm$ 0.20, for C - 1.35 $\pm$ 0.05, and for D - 0.07 $\pm$ 0.08.

Table 5: *Values of empirical coefficients in the parameterization formula for the best fit for the airport stations for July 2012.*

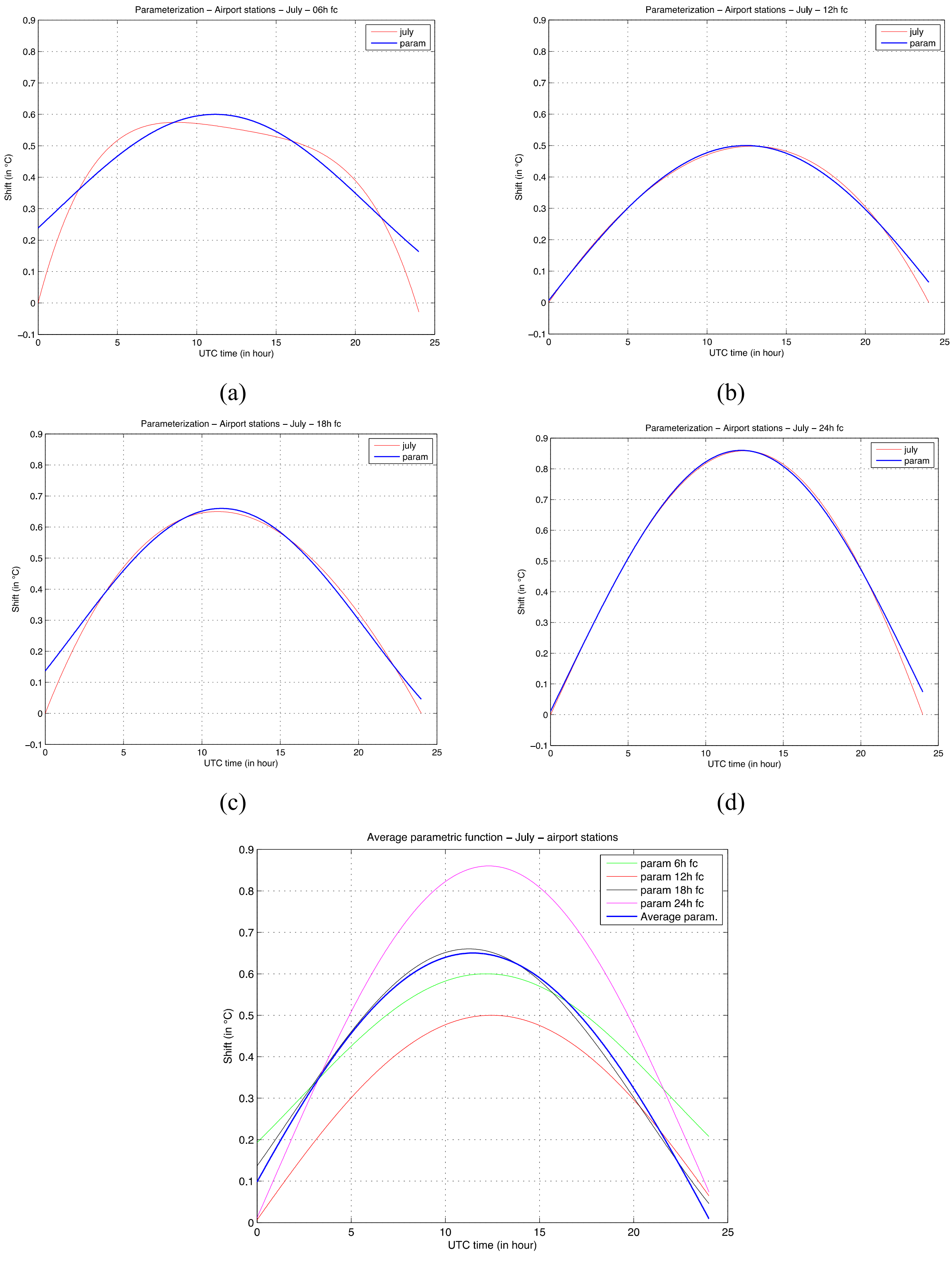
Reference to Figure	Forecast length	Empirical coefficients			
		A	B	C	D
13a	06h	0.30	30.9	2.0	0.30
13b	12h	0.55	39.1	2.7	-0.05
13c	18h	0.40	30.0	1.9	0.26
13d	24h	0.75	35.0	2.3	0.11
13e	***Overall averaged***	***0.75***	***39.1***	***2.8***	***-0.10***

An interval was also calculated for these coefficients: A is changing 0.50 $\pm$ 0.25, for B - 35 $\pm$ 5, for C – 2.3 $\pm$ 0.4, and for D - 0.10 $\pm$ 0.15.



(e)

Figure 12: *Parameterization of T2m error shift for each forecast length: (a) 6h, (b) 12h (c) 18h, and (d) 24h; and (e) Averaged parametric functions for different forecast lengths and overall (6-12-18-24h) averaged - for the AS group of stations for May 2012.*



(e)

Figure 13: *Parameterization of T2m error shift for each forecast length: (a) 6h, (b) 12h (c) 18h, and (d) 24h; and (e) Averaged parametric functions for different forecast lengths and overall (6-12-18-24h) averaged - for the AS group of stations for July 2012.*

3.4. Improvements for BIAS and MAE of 2m air temperature forecast

3.4.1. High error stations

The parameterization used in this section is the overall (include 06-12-18-24 times) averaged parametric function (see Figure 11(e) in section 3.3.1) for the HES group of stations:

$$S(t) = S(t_0) + 0.800\cos\left(\pi + \frac{t - 22.65}{1.10\pi}\right) + 0.65 \quad (9)$$

with $S(t_0)$ the monthly averaged T2m error shift at midnight (UTC time).

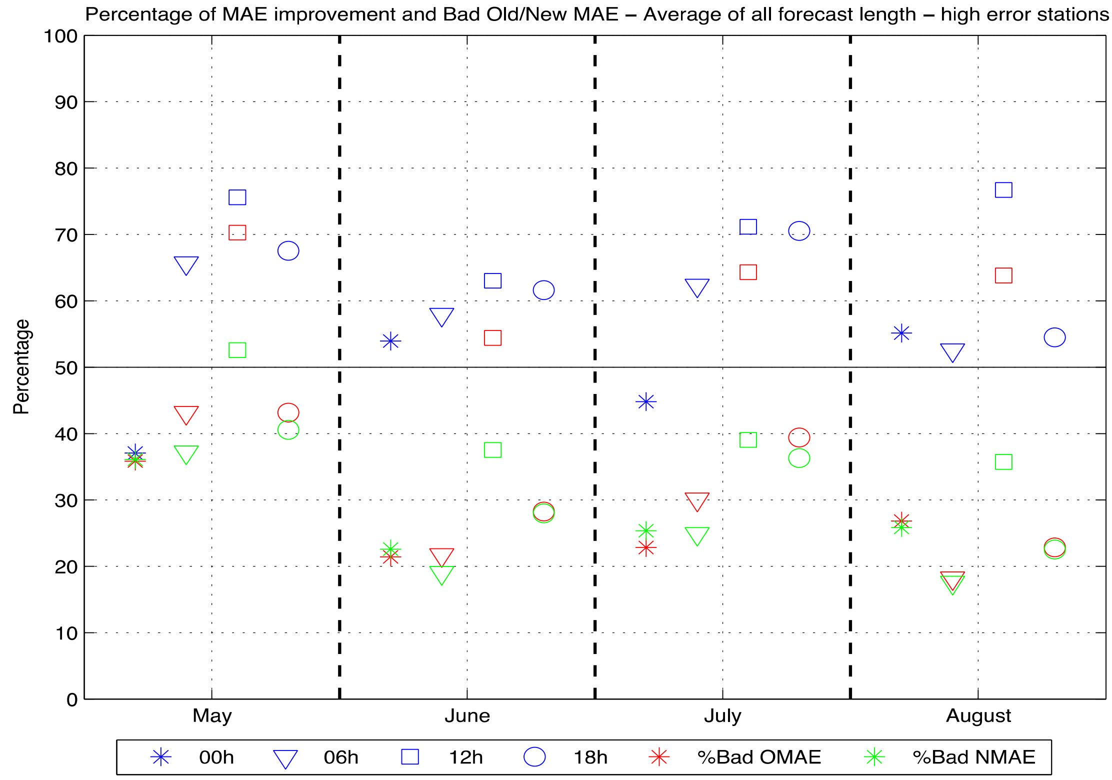
Figures 14a, 14b and 14c show the improvement of the two key indicators: BIAS and MAE became better by the correction of 2m air temperature for all forecast lengths (06h, 12h, 18h and 24h). Results for each forecast length are given the Appendix B.1. Figure 14a presents the percentage of the MAE improvement for each time of the day (00h, 06h, 12h and 18h in UTC time) for all months (May, June, July and August 2012). The old MAE (before the correction) and the new MAE (after the correction) are calculated for each observation. The results are shown by blue color symbols. The red symbols present the percentage of old MAE higher than 1°C and the green symbols present the percentage of new MAE higher than 1°C. Figures 14b and 14c show the comparison between the old and new values of the monthly BIASes and monthly MAEs calculated for each time of the day. Table 6 summarizes the results from Figure 14 for each time of the day.

Figure 14 shows that the corrections are the most efficient for three (of four) times of the day: at 06h, 12h and 18h (UTC time) for all months (from May to August 2012). For these, the BIAS and the MAE are reduced. Moreover, the correction is the largest for the mid-day. The peaks of the BIAS and MAE observed at mid-day time on Figures 14b and 14c have a significant improvement (almost by 0.5°C). Furthermore, at this time, there are about 20% of MAE which become improved or less than 1°C (see Table 6). However, for May and July 2012 at midnight, the monthly BIAS and MAE are less improved comparing with other times.

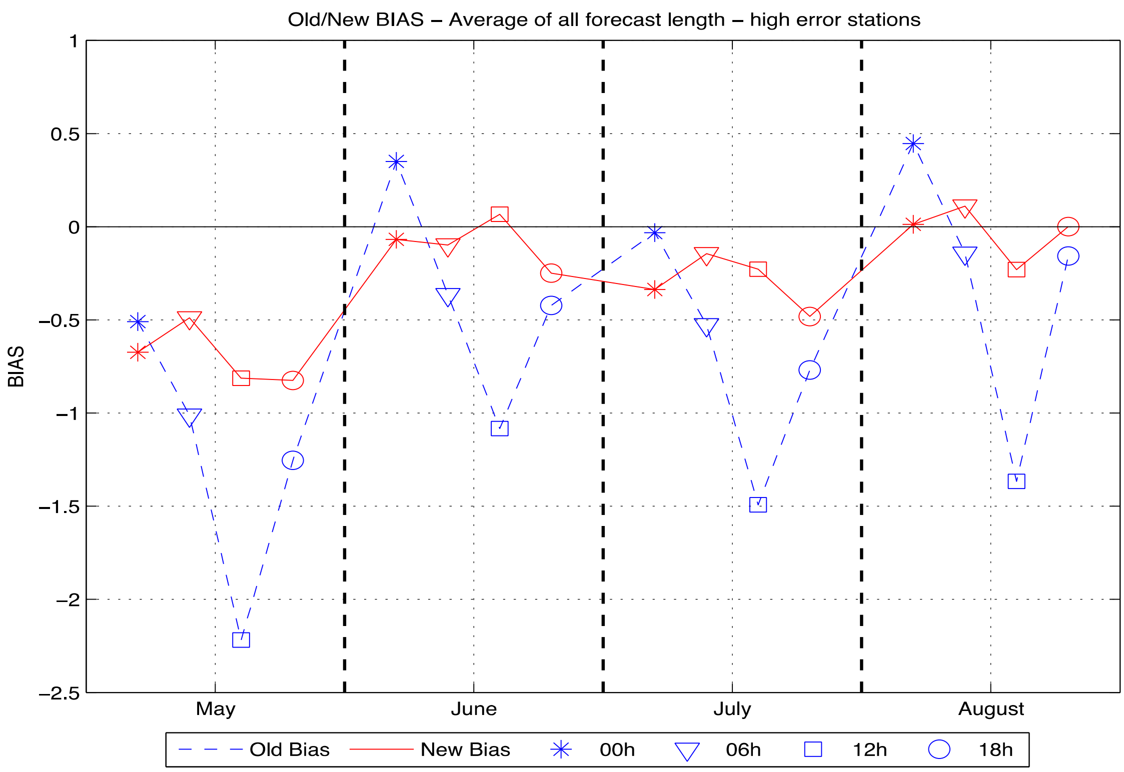
Table 6: *Averaged percentage per time of day for months (May-Jun-Jul-Aug 2012) with the improvement to the 2m air temperature forecast for all forecast lengths for the HES group of stations.*

Time of the day (in UTCs)	Percentage of				
	improved cases for MAE	cases for MAE > 1		improved for monthly	
		before correction	after correction	BIAS	MAE
00h	47.2	26.9	27.7	50	50
06h	60.0	29.0	25.2	75	100
12h	71.7	63.5	41.7	100	100
18h	64.1	34.2	32.5	100	100
Overall averaged	***60.8***	***38.4***	***31.8***	***87***	***87***

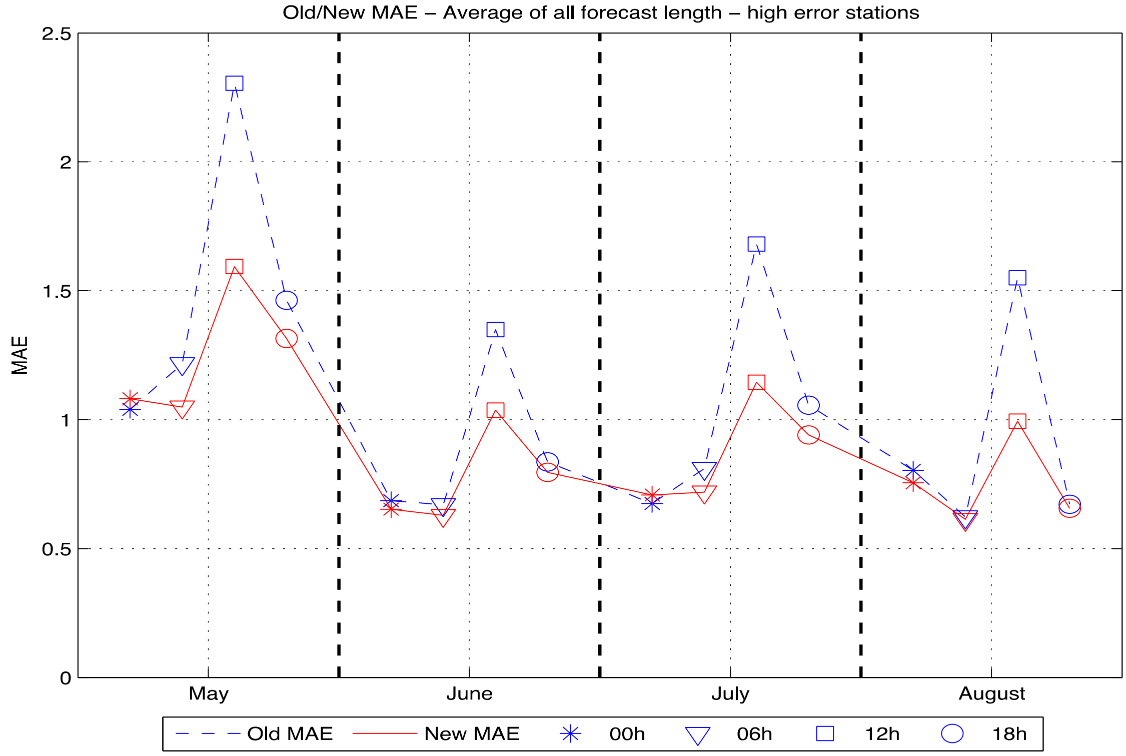
Finally, in total, 61% of cases (Table 6) have improved the error of the 2m air temperature. Note, referring to the Appendix C (see Table C.2) the number of cases is summarized. For the monthly BIAS (see Figure 14b), it is improved for about 87% (see Table 6). There is the same result for the monthly MAE (see Figure 14c). Moreover, Figure 14c shows general tendency that it became more complex to improve MAE values which became smaller (i.e. less than 1°C).



(a) In blue : Percentage of MAE improvement (Old MAE higher than New MAE) – In red : percentage of old MAE higher than 1°C – In green : percentage of new MAE higher than 1°C for ***HES group***

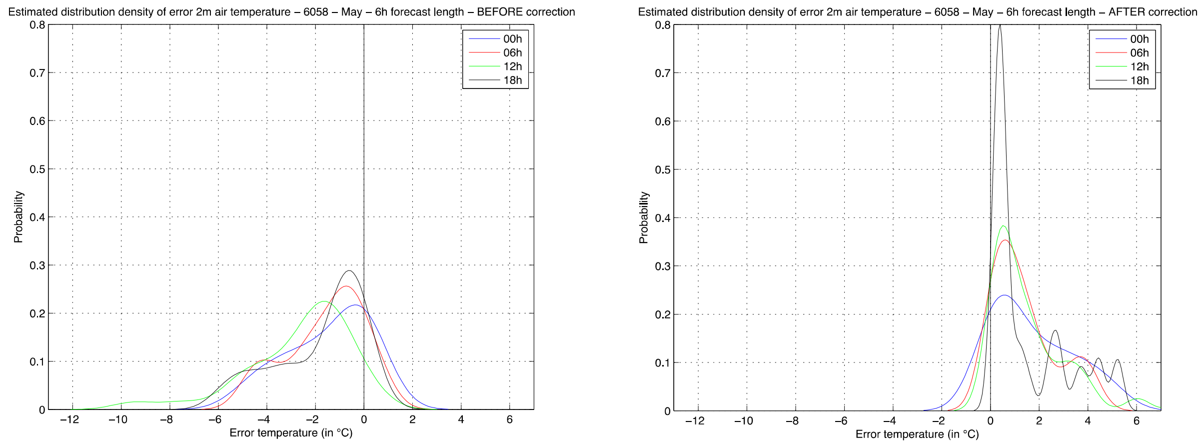


(b) Old and New monthly BIAS for each time (in symbols) of the day for all forecast lengths – ***HES group***



(c) Old and New monthly MAE for each time (in symbols) of the day - All forecast length – ***HES group***

Figure 14: *Summary of results on improvement for percentage, BIAS and MAE for the 2m air temperature forecasts for all forecast lengths for the HES group of stations.*



(a) *(b)*

Figure 15 *: Estimated distribution density of 2m air temperature error for all period of the day for station 6058 (HES) in May 2012 for 6h forecast length (a) before and (b) after correction.*

Figure 15 shows an example of the estimated distribution density for T2m error for all times of the day in May 2012 for the HES station WMO-06058 before and after correction of the temperature. It is seen that the peaks of distributions for all times became close to 0-line, and especially closer for 18h (in black). The distribution of mid-night values is approximately the same in error. As seen, the larger number of cases (with revised forecasts) has smaller biases, and the range of biases (min vs. max bias) was also improved (or shortened).

3.4.2. Airport stations

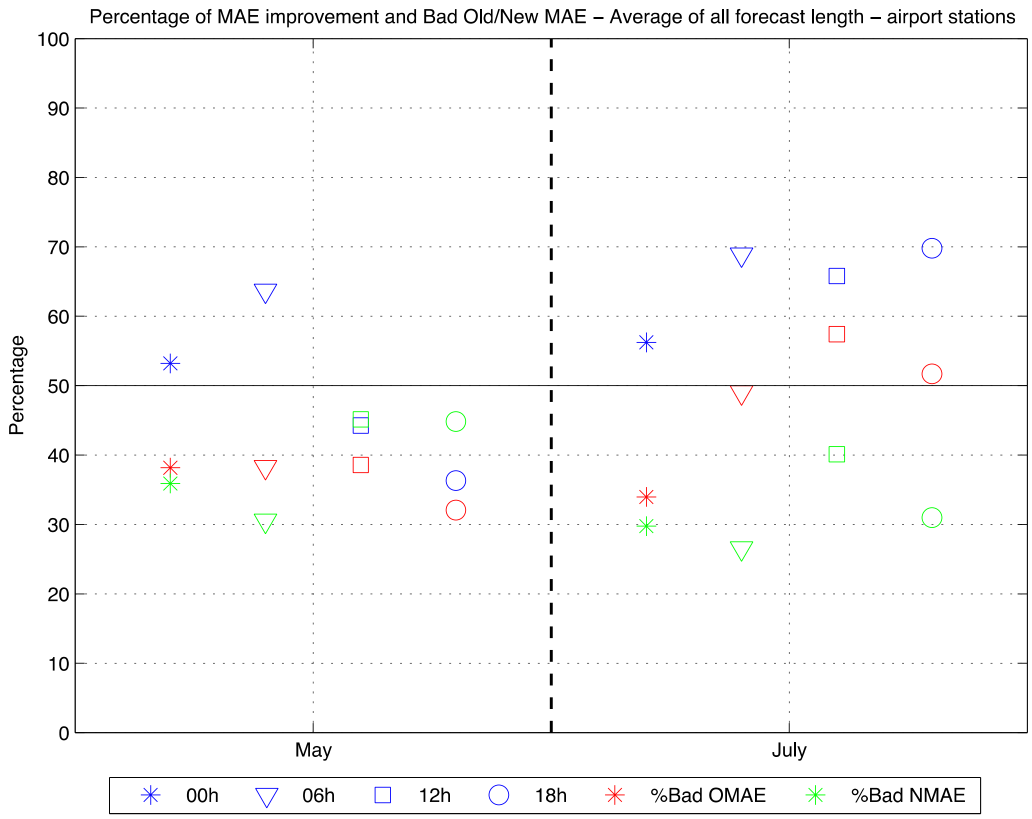
The parameterization used in this section is the overall (include 06-12-18-24 times) averaged parametric function for May/ July (see Figure 12e/13e in section 3.3.2) for the AS group of stations:

$$S_{may}(t) = S(t_0) + 0.25\cos\left(\pi + \frac{t-18.3}{1.3\pi}\right) + 0.05 \qquad (10)$$

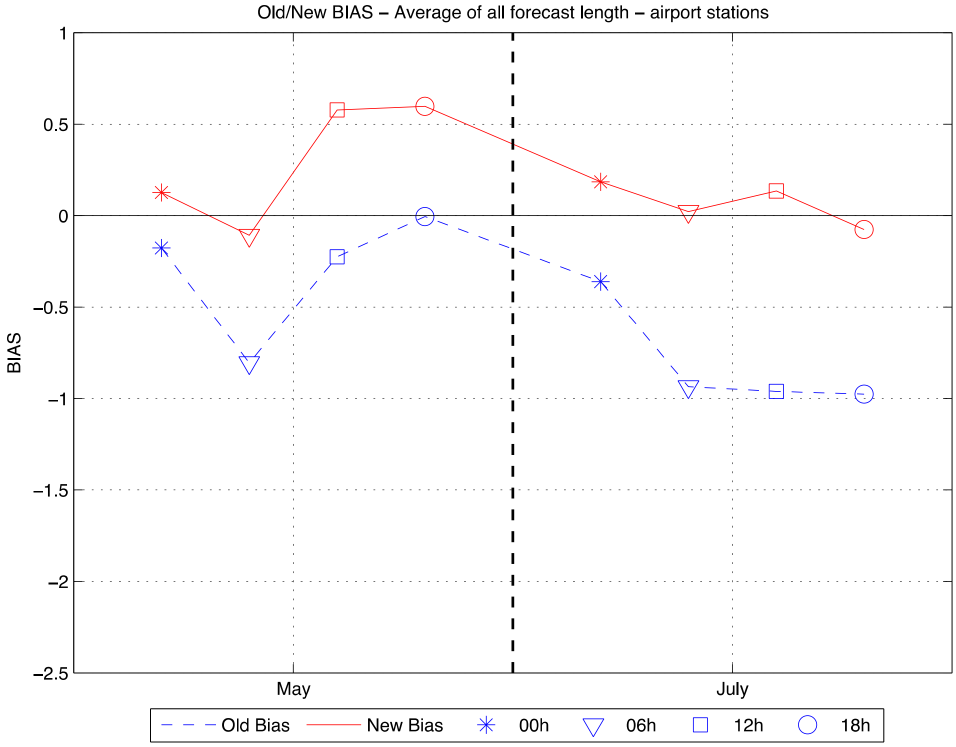
$$S_{jul}(t) = S(t_0) + 0.75\cos\left(\pi + \frac{t-39.1}{2.8\pi}\right) - 0.10 \qquad (11)$$

where $S(t_0)$ is the monthly average shift at midnight (in UTC time).

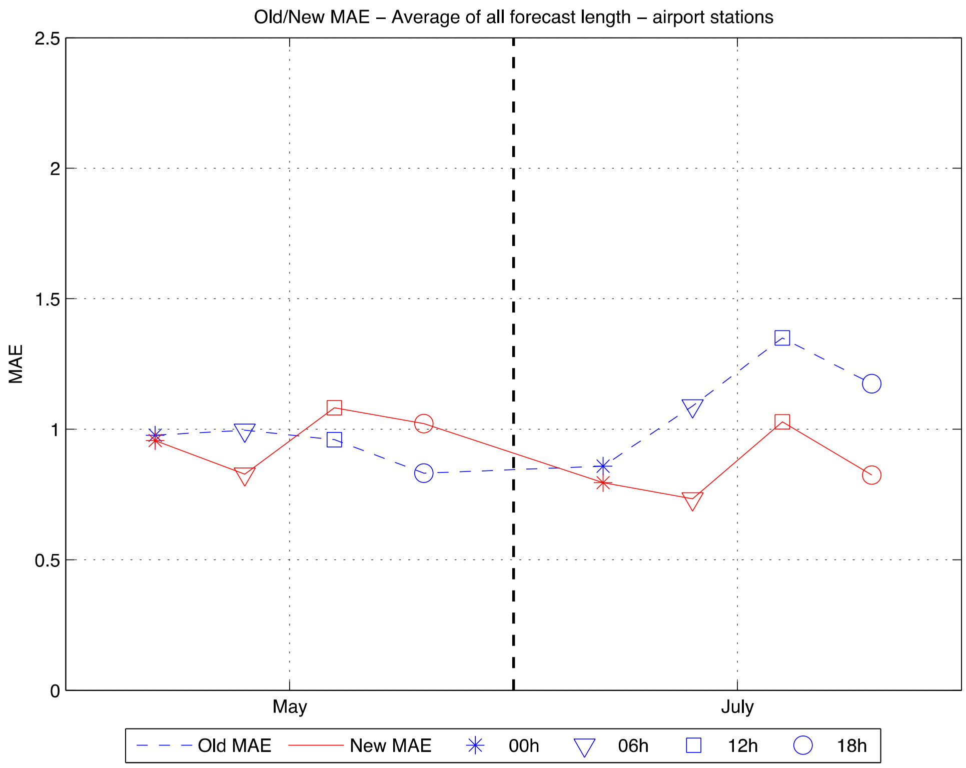
Figure 16 shows the improvement of the BIAS and MAE after correction of 2m air temperature forecast. In Figure 16a, we can observe that the correction is more efficient for 06h (UTC time) for May and July 2012. About 66.3% cases for MAE have improved during this time of the day (see Table 7). In July 2012, the number of cases with new MAE higher than 1°C has decreased by almost 20% (Figure 16a). However, in daytime (12h and 18h in UTC time) in May 2012, we can observe an increase of the monthly BIAS and MAE values (by an extra 0.6°C for the BIAS and by extra 0.2°C for the MAE). Moreover, the number of cases with new MAE higher than 1°C has increased by 10%.
Figure 16(a-b-c) shows that the correction is more efficient in July 2012 compared with May 2012. In July 2012 for all cases the monthly BIAS and MAE have improved, while in May it appeared just in daytime. The new BIAS became closer to 0°C for all times of the day in July. Finally, there are about 57.3% of cases in May and July, when the MAE was also improved (see Table 7).



*(a) In blue : Percentage of MAE improvement (Old MAE higher than New MAE) – In red : percentage of old MAE higher than 1°C – In green : percentage of new MAE higher than 1°C - **AS group***



*(a) Old and New monthly MAE for each time (in symbols) of the day on May and July 2012 - All forecast lengths – **AS group***



*(b) Old and New monthly MAE for each time (in symbols) of the day on May and July 2012 - All forecast lengths – **AS group***

Figure 16: *Summary of results on improvement for percentage, BIAS and MAE for the 2m air temperature forecasts for all forecast length for the AS group of stations.*

***Table 7:** Averaged percentage per time of day for selected months (May and July 2012) with the improvement to the 2m air temperature forecast for all forecast lengths for the AS group of stations.*

Time of the day (in UTCs)	Percentage of				
	improved cases for MAE	cases for MAE > 1		improved for monthly	
		before correction	after correction	BIAS	MAE
00h	54.7	36.1	32.8	100	100
06h	66.3	43.7	28.5	100	100
12h	55.1	48.1	42.6	50	50
18h	53.1	42.0	37.9	50	50
Overall averaged	***57.3***	***42.5***	***35.5***	***75***	***75***

4. Conclusion

Detailed analysis of observations and forecasts of four meteorological parameters (2m air temperature, dew temperature, surface temperature, and wind speed) had been realized for May-Aug 2012 dataset. A strong correlation between temperatures was identified for two groups of stations: 1) high error stations (HES) and airport stations (AS). It was found that each selected meteorological parameter has a distribution density close to the normal; and all these parameters have a periodicity on a diurnal cycle. In this study, a parameterization for the improvement of the air temperature forecast based on estimation of an average shift of the distribution density of the 2m air temperature error was elaborated, tested and verified for different times of the day, different forecast lengths and different month of the studied period. The developed parameterization was tested for individual and groups of stations. It showed improvement and good agreement with observations for the HES group of stations for all times and forecast lengths in all months. And for AS stations the improvement was more visible in May and Jul 212. The developed parameterization can be implemented for operational tasks of numerical weather prediction.

Acknowledgments

The financial support for research training and study was provided by the INSA (Institut National des Sciences Appliquees, Toulouse, France) and Conseil Regional du Midi-Pyrenees (Toulouse, France). Thanks to DMI researchers for constructive comments and discussions.
The DMI computing facilities and advising were provided. Thanks to DMI's Computer Department for advice. The research has been carried out using MATLAB.

References

Gilet N., Mahura A., Petersen C., Sass B. (**2013**): Analysis and Improvement of Statistical Error in Forecasted Air Temperature. *DMI Scientific Report 13-03, ISBN: 978-87-7478-646-7, 39p,* www.dmi.dk/dmi/sr13-03.pdf

Hartmann D. L. (**2013**). Basic course in atmospheric sciences statistical analysis; ATM – 552: Objective Analysis. *Review of Statistics. http://www.atmos.washington.edu/~dennis/552_Notes_1.pdf*

Mahura A., Baklanov A., Korsholm U. (**2009**). Parameterization of the birch pollen diurnal cycle. *Aerobiologia 25, 203-208.*

Mahura A., Petersen C., Sass B., Gilet N. (**2013**): Statistical Correction of Air Temperature Forecasts for City and Road Weather Applications. *DMI Scientific Report 13-02, ISBN: 978-87-7478-645-0, 49p,* www.dmi.dk/dmi/sr13-02.pdf

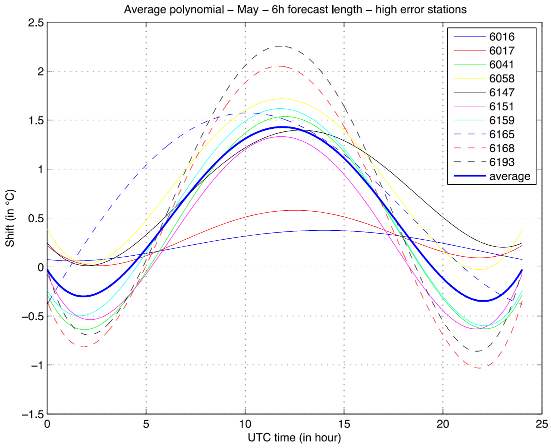
Petterssen S. (**1969**). Introduction to Meteorology

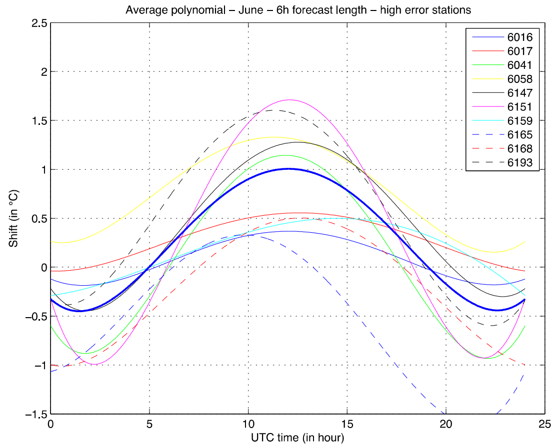
Wallace J.M. and Hobbs P.V. (**1977**). Atmospheric Science, an introduction survey

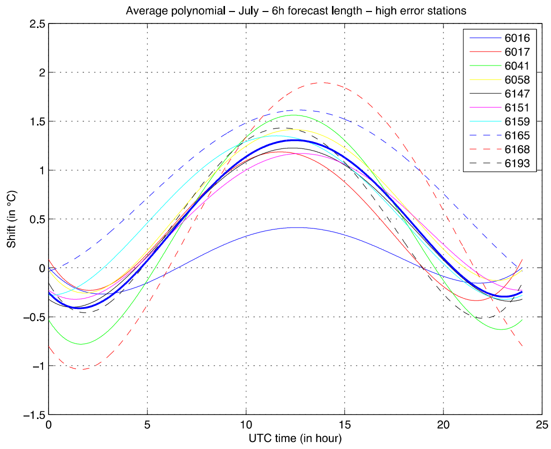
Appendix A: Averaged polynomial fit to air temperature error function

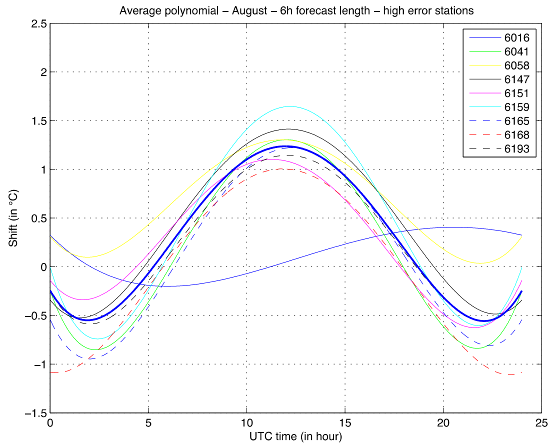
A1. For high error stations

A1.1. For 6h forecast length

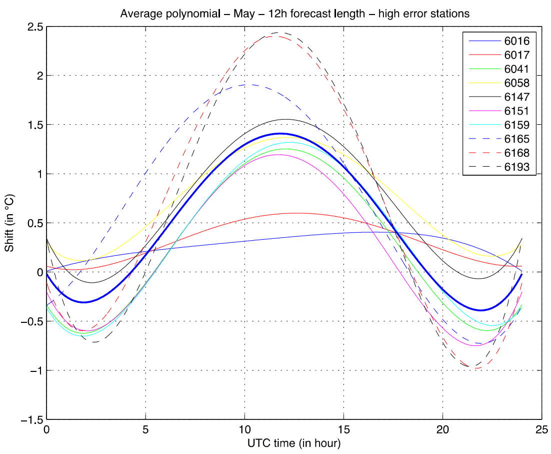


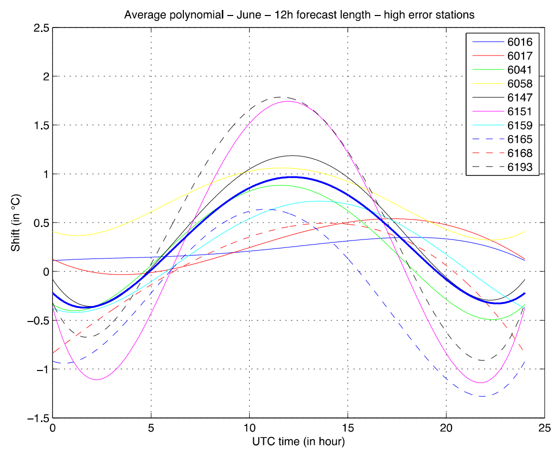


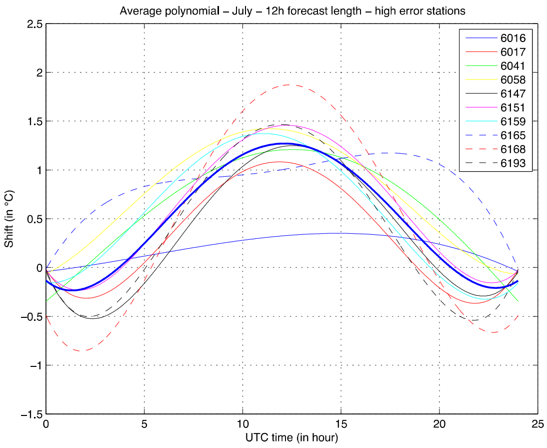


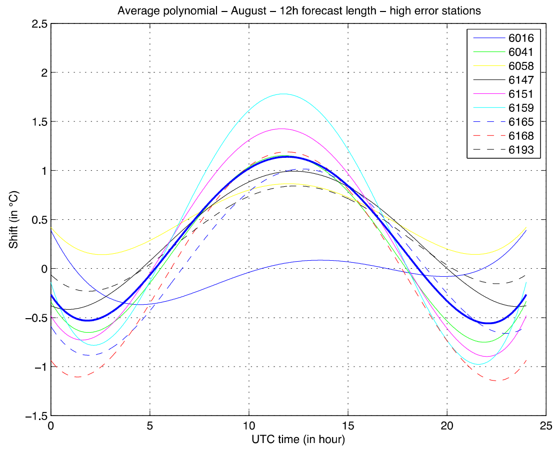


A1.2. For 12h forecast length

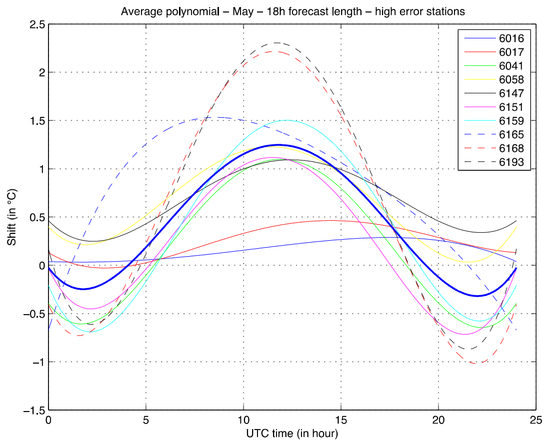


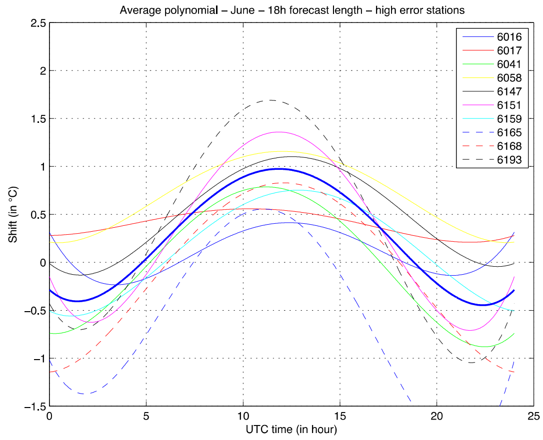


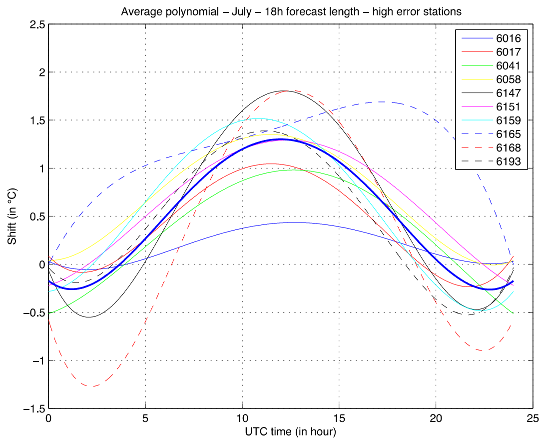


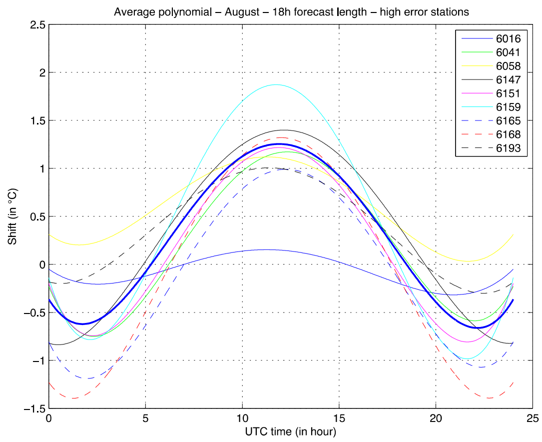


A1.3. For 18h forecast length

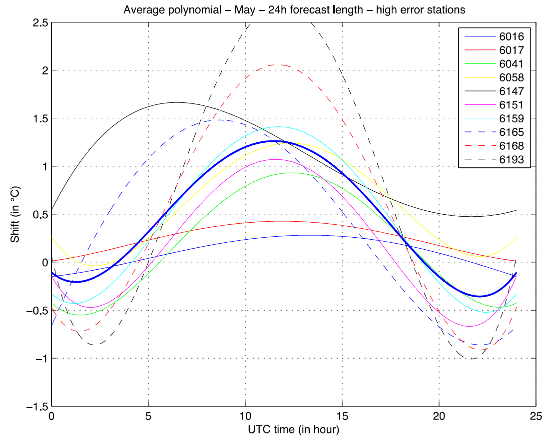


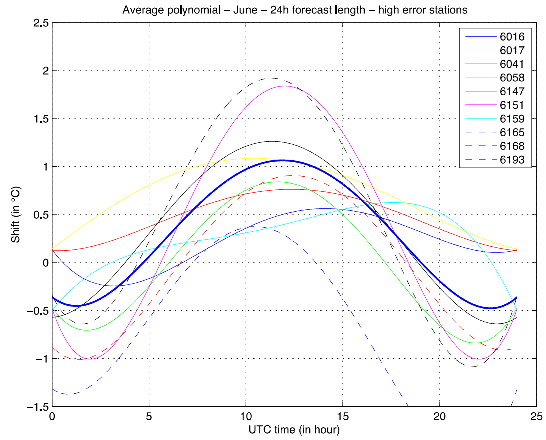


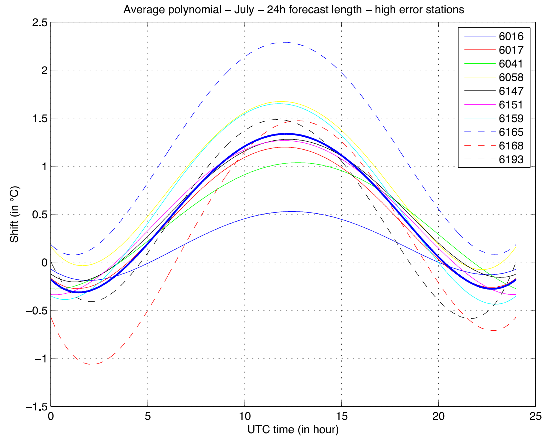


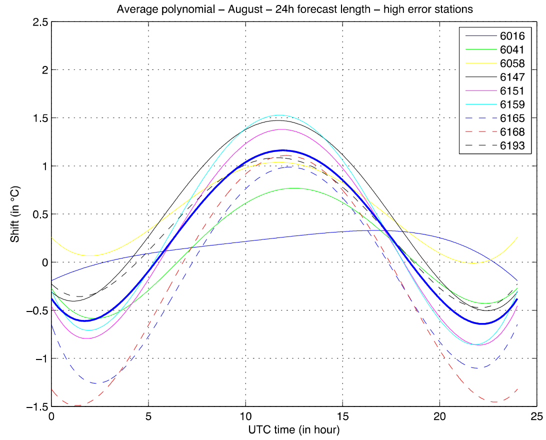


A1.4. For 24h forecast length



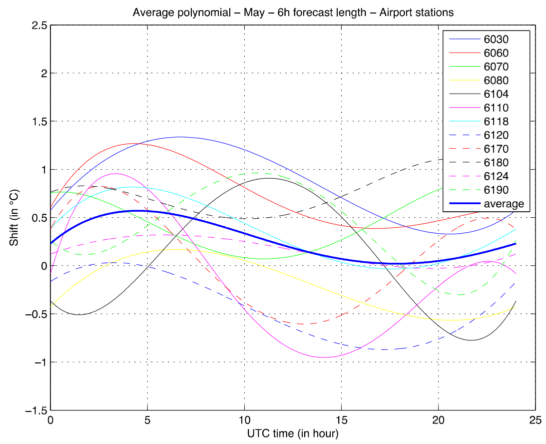


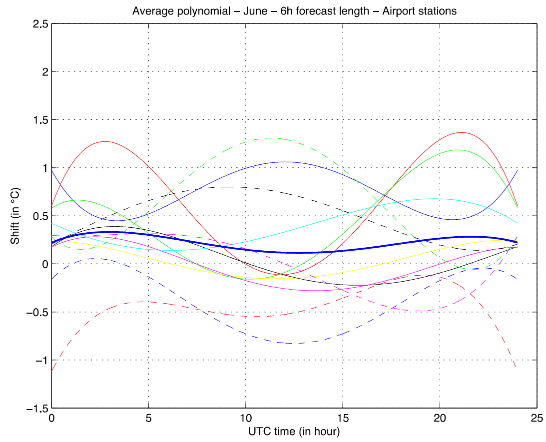


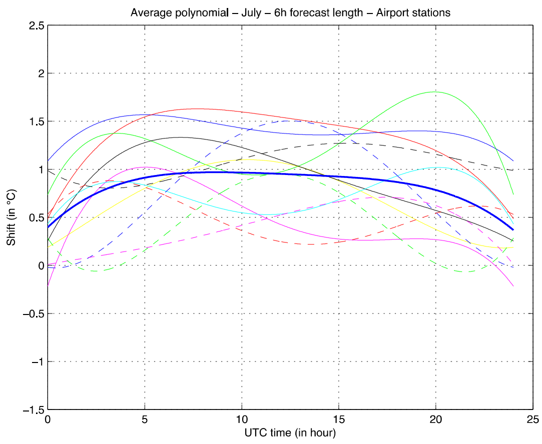


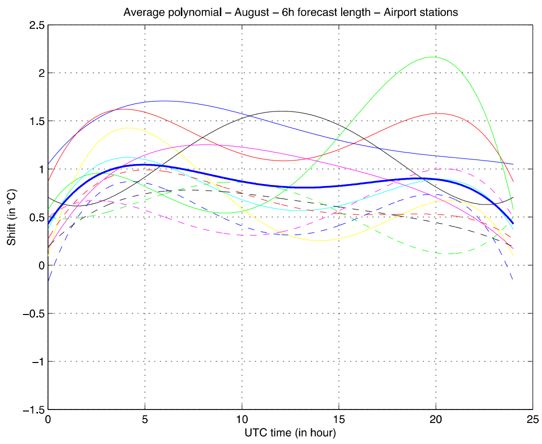
A2. For airport stations

A2.1. For 6h forecast length

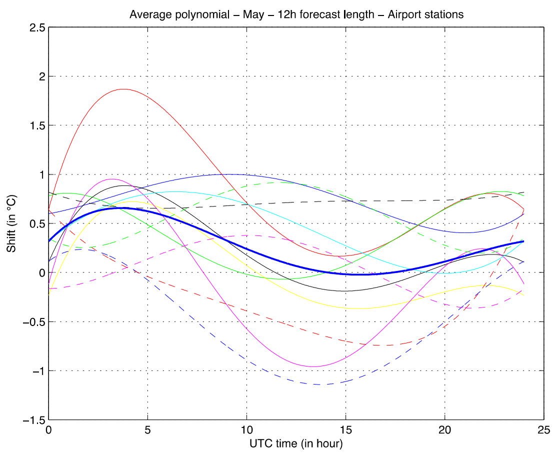


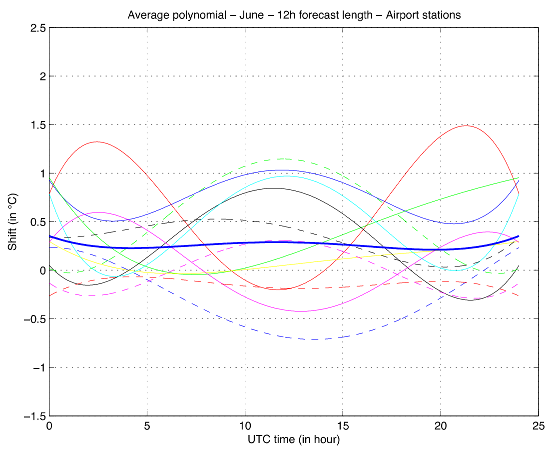


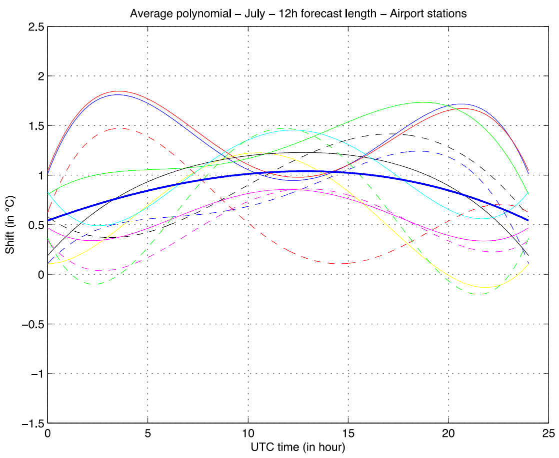


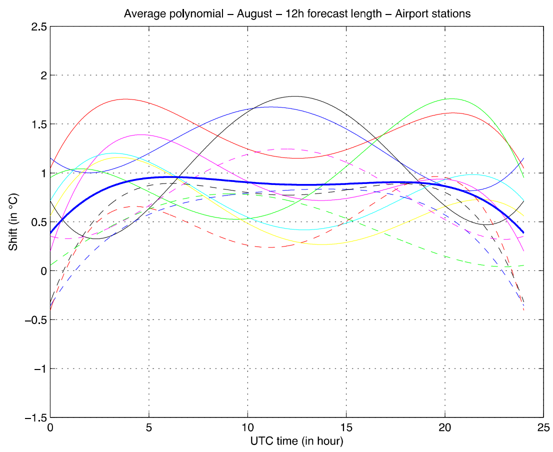


A2.2. For 12h forecast length

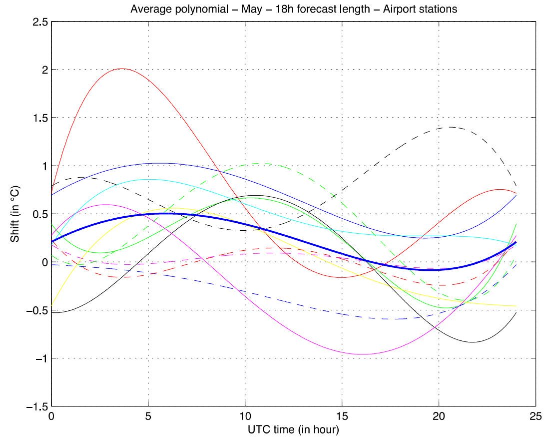


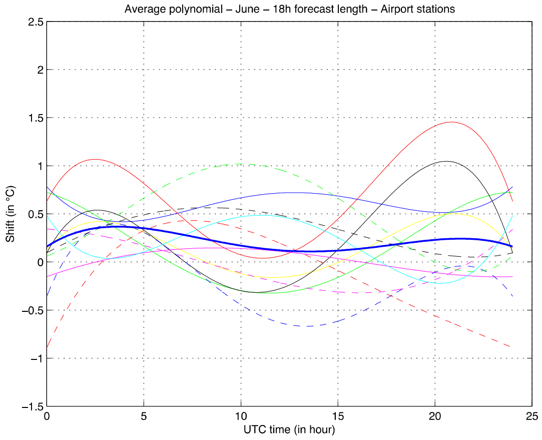


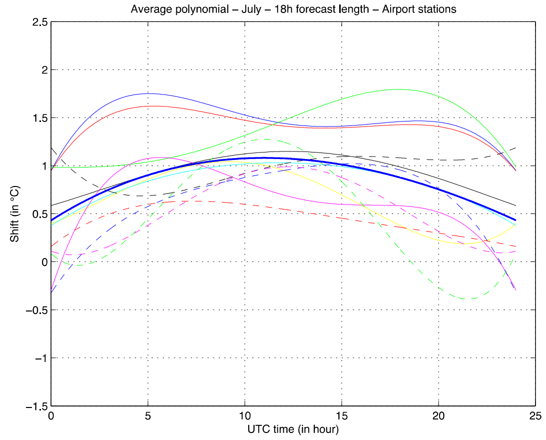


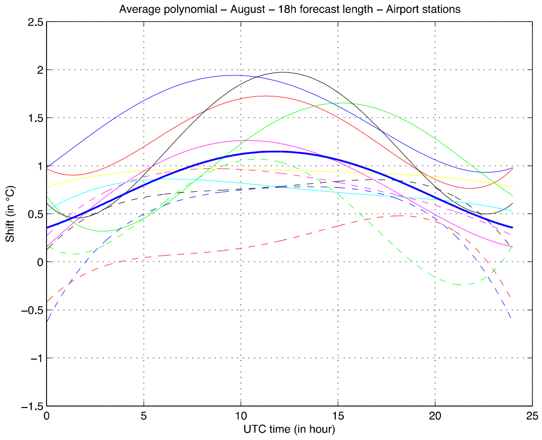


A2.3. For 18h forecast length

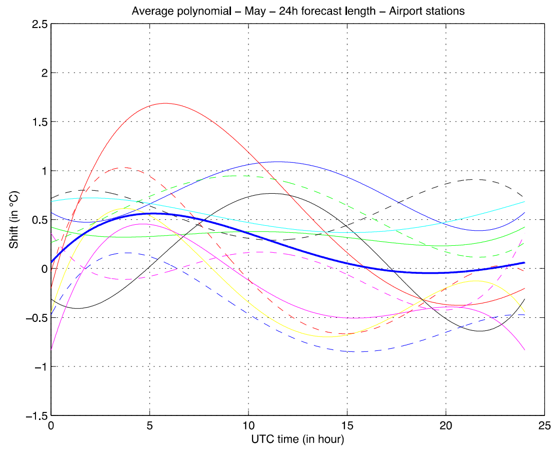


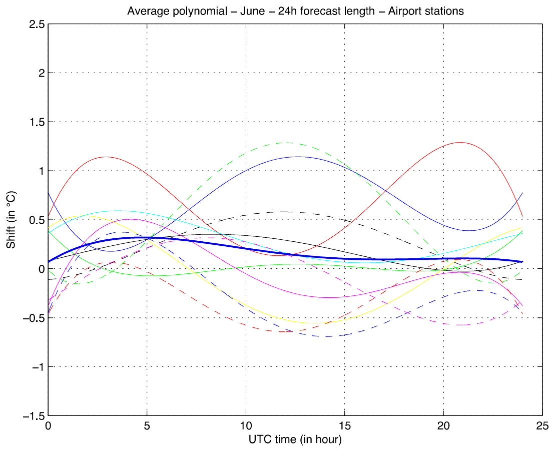


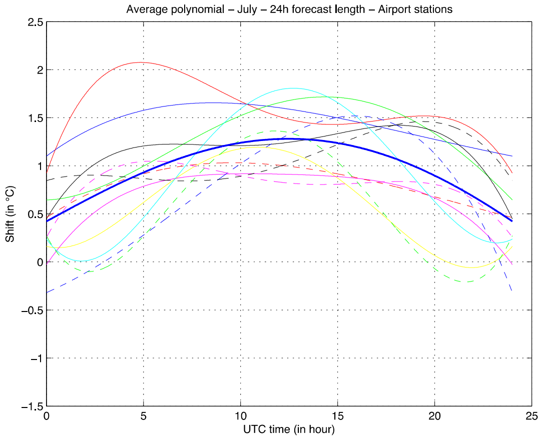


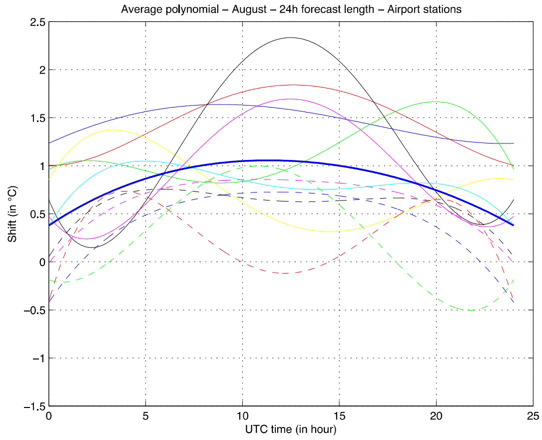


A2.4. For 24h forecast length





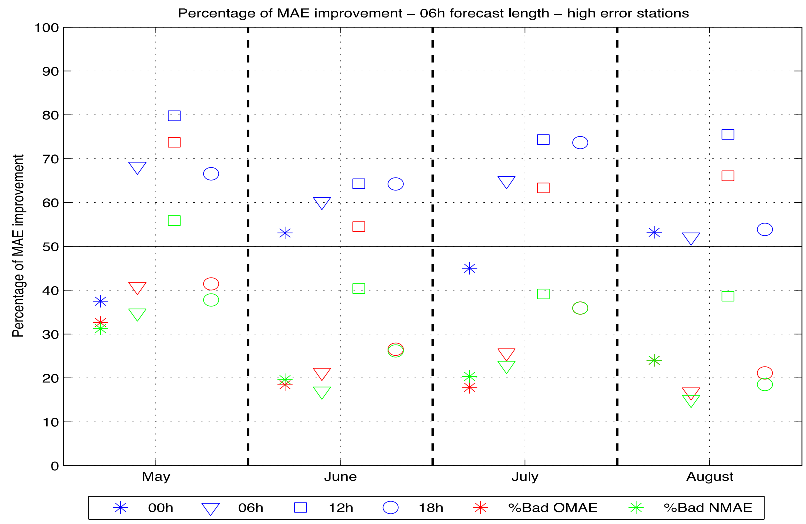
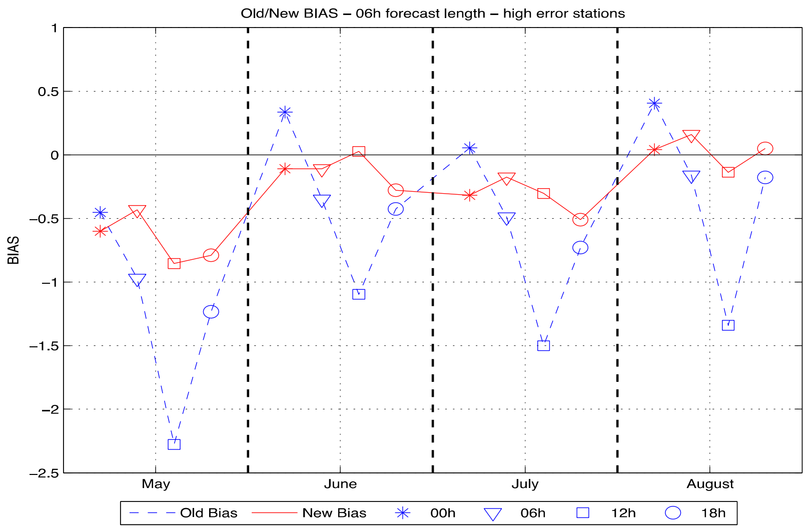
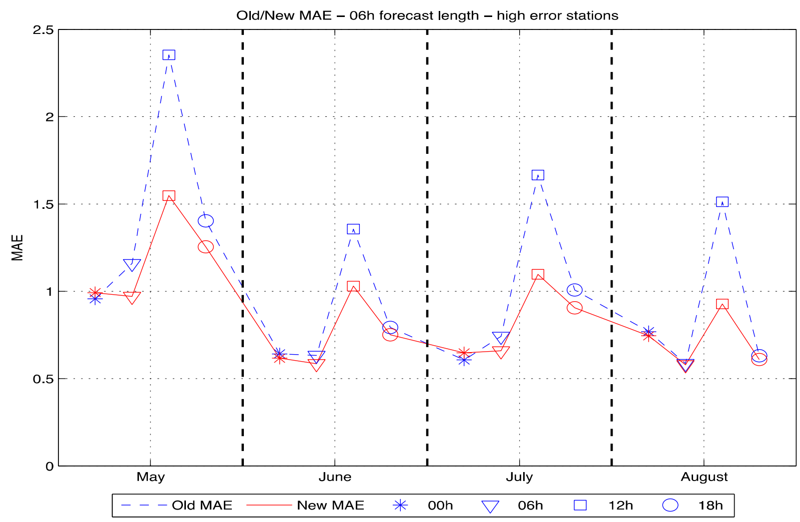

Average polynomial – July – 24h forecast length – Airport stations
Shift (in °C)
UTC time (in hour)
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
0
5
10
15
20
25


Average polynomial – August – 24h forecast length – Airport stations
Shift (in °C)
UTC time (in hour)
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
0
5
10
15
20
25

Appendix B: Improvement of air temperature forecast (percentage, BIAS, MAE)

B1. High error stations

B1.1. For 6h forecast length

In blue: Percentage of MAE improvement *In red: percentage of Old MAE >1* *In Green: percentage of New MAE >1* *06h forecast length – HES group*	
Old and New monthly BIAS for each time of the day *06h forecast length – HES group*	
Old and New monthly MAE for each time of the day *06h forecast length – HES group*	

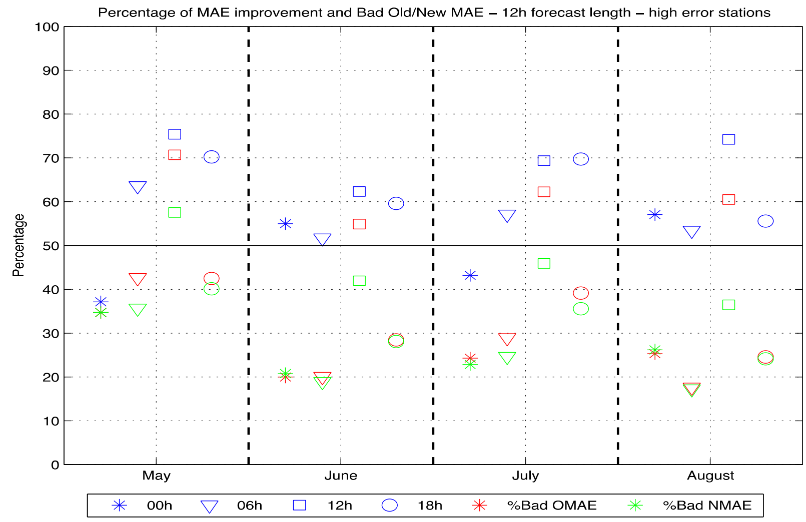
B1.2. For 12h forecast length

In blue: Percentage of MAE improvement

In red: percentage of Old MAE >1

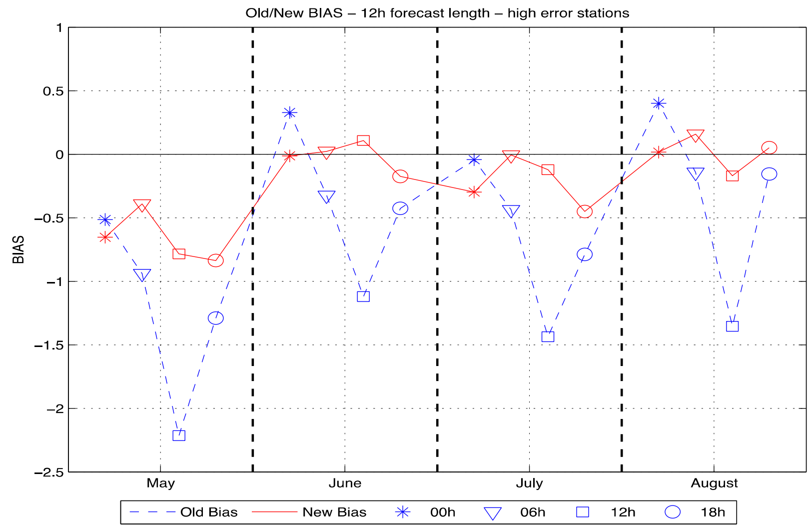
In Green: percentage of New MAE >1

12h forecast length – HES group



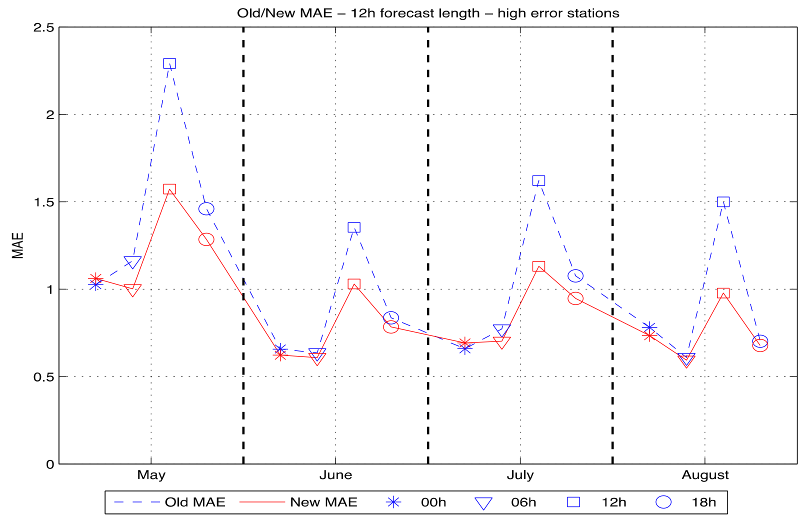
Old and New monthly BIAS for each time of the day

12h forecast length – HES group

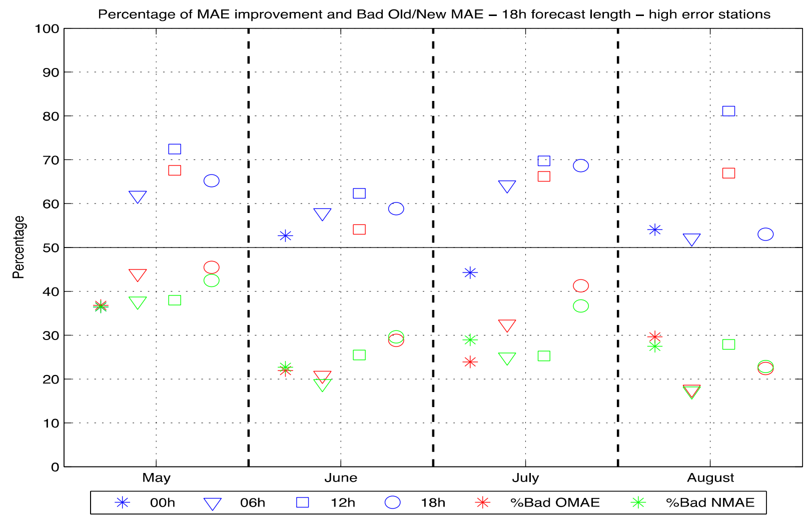
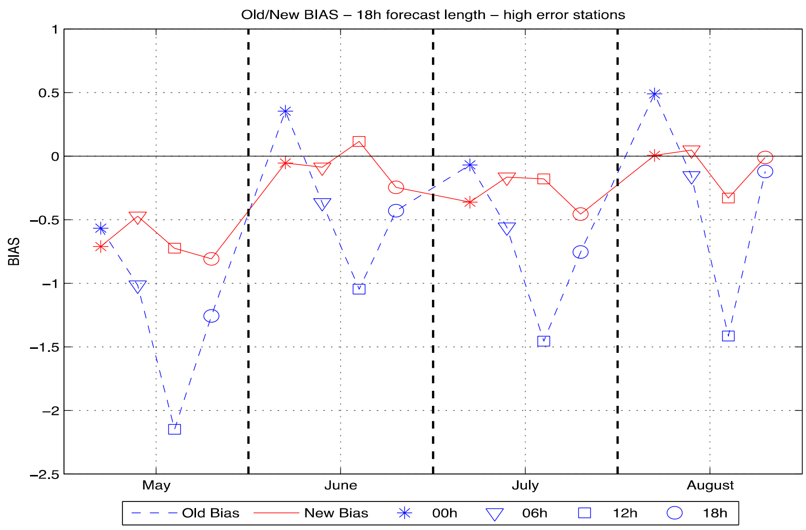
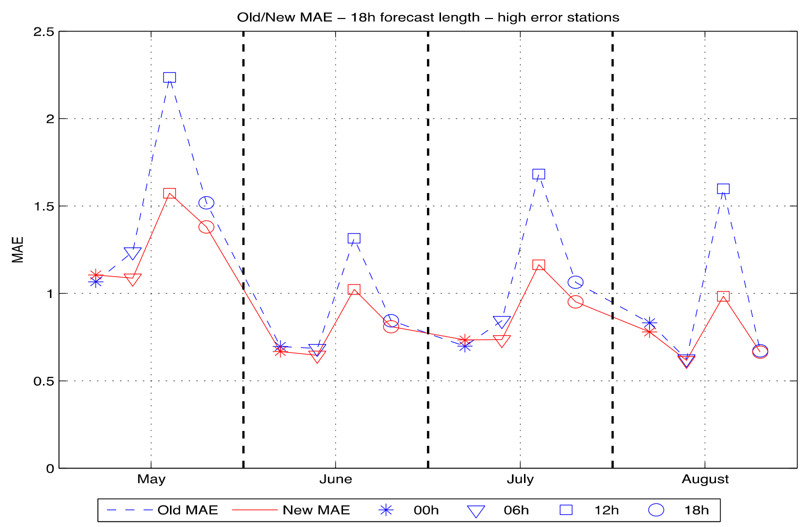


Old and New monthly MAE for each time of the day

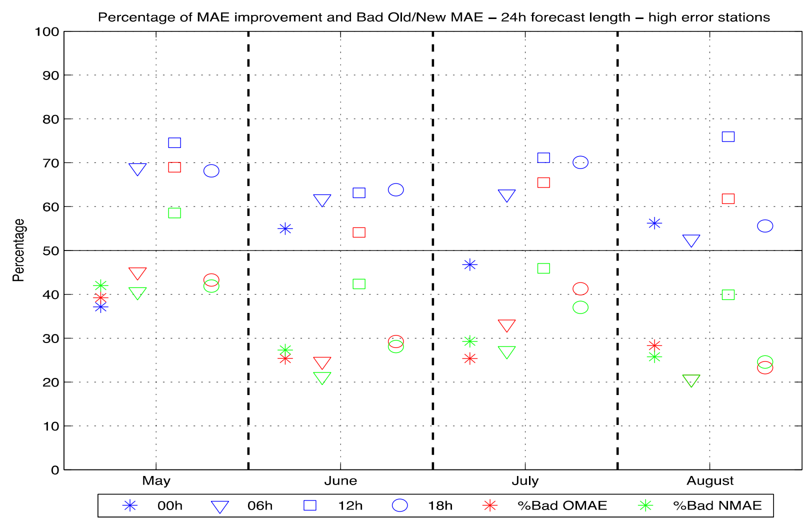
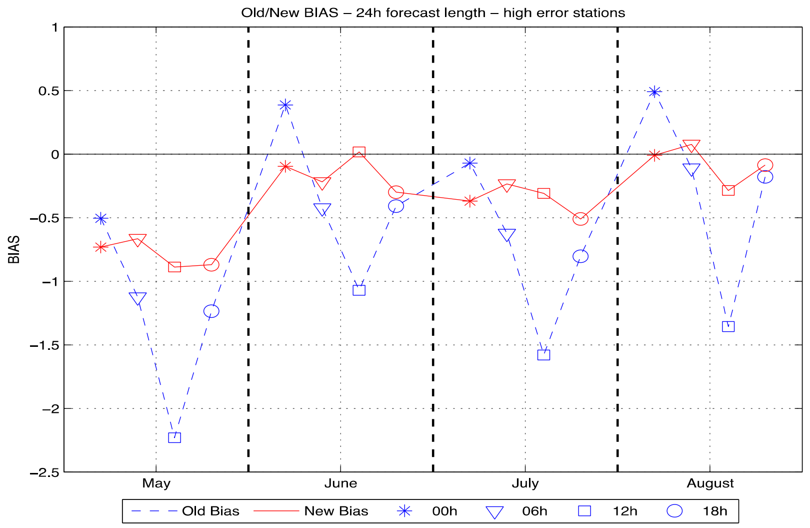
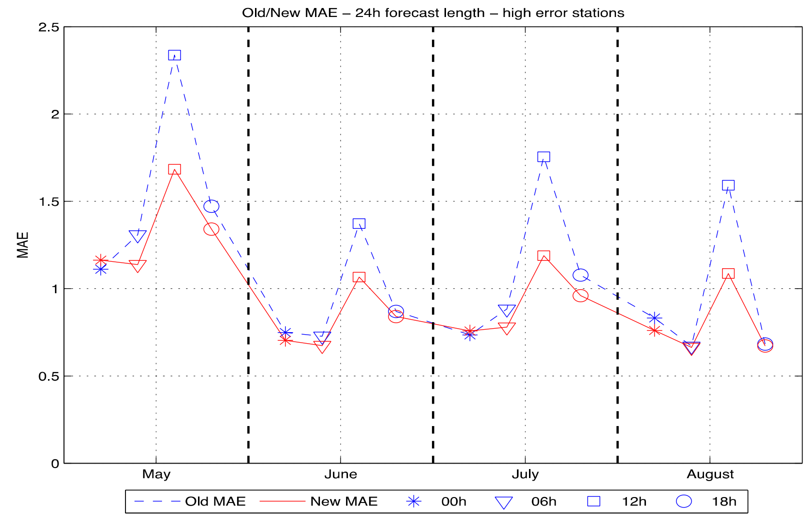
12h forecast length – HES group



B1.3. For 18h forecast length

In blue: Percentage of MAE improvement *In red: percentage of Old MAE >1* *In Green: percentage of New MAE >1* *18h forecast length – HES group*	
Old and New monthly BIAS for each time of the day *18h forecast length – HES group*	
Old and New monthly MAE for each time of the day *18h forecast length – HES group*	

B1.4. For 24h forecast length

In blue: Percentage of MAE improvement *In red: percentage of Old MAE >1* *In Green: percentage of New MAE >1* *24h forecast length – HES group*	
Old and New monthly BIAS for each time of the day *24h forecast length – HES group*	
Old and New monthly MAE for each time of the day *24h forecast length – HES*	

B2. Airport stations

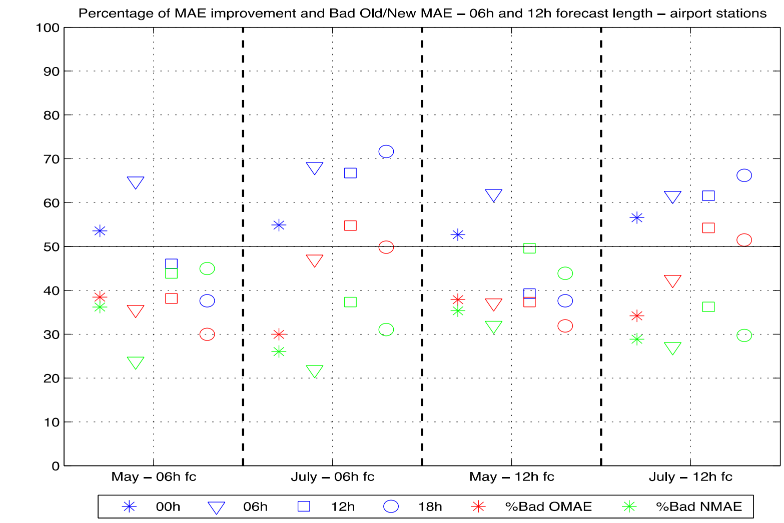
B2.1. For 6h and 12h forecast lengths

In blue: Percentage of MAE improvement

In red: percentage of Old MAE >1

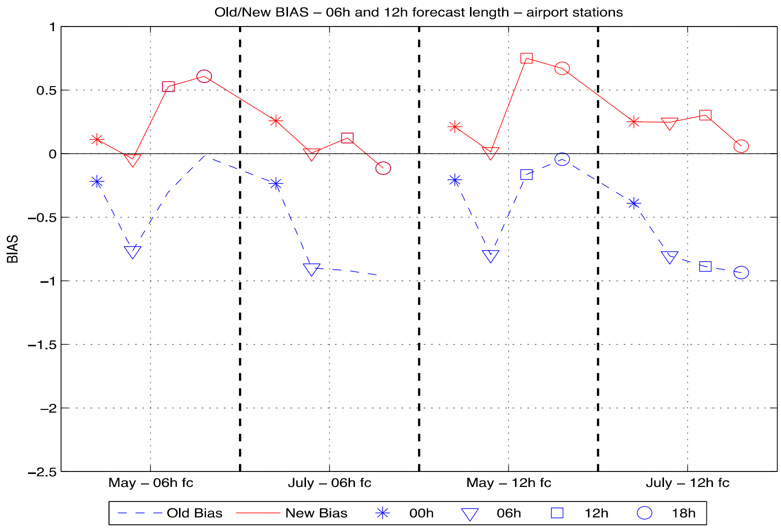
In Green: percentage of New MAE >1

06h and 12h forecast length – AS group



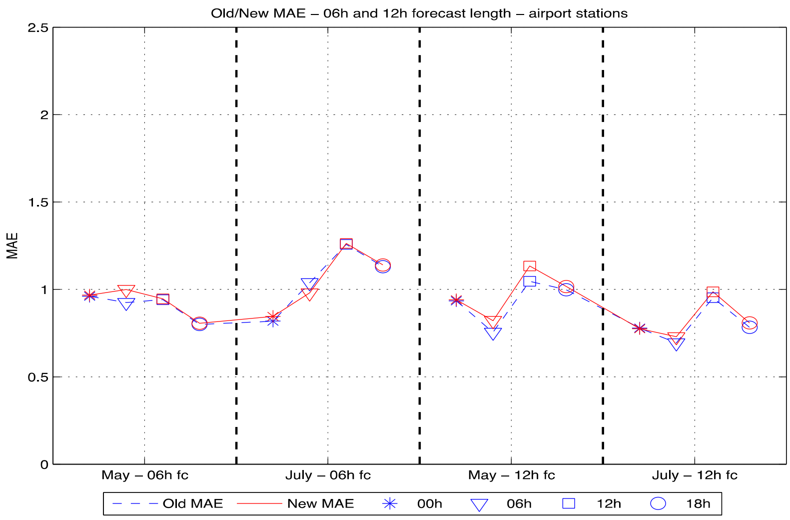
Old and New monthly BIAS for each time of the day

06h and 12h forecast length – AS group

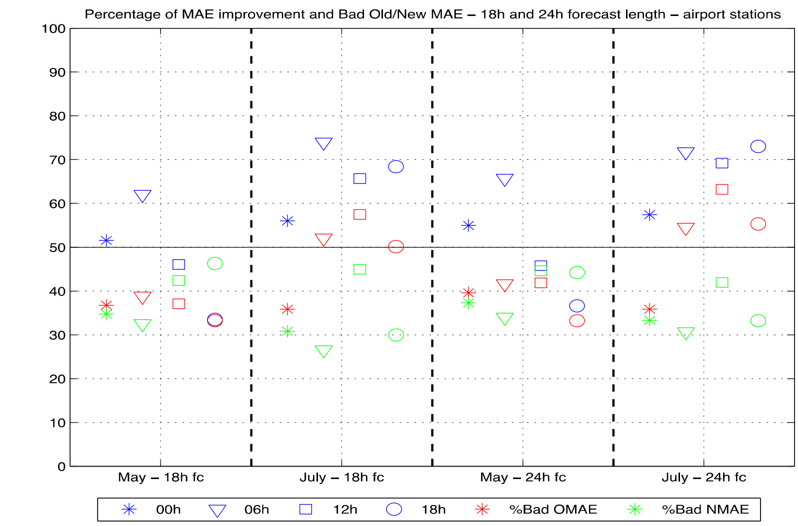
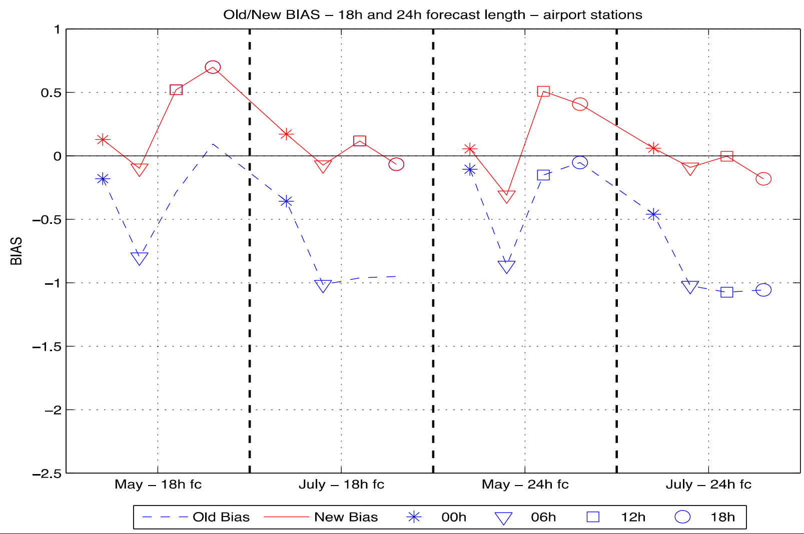
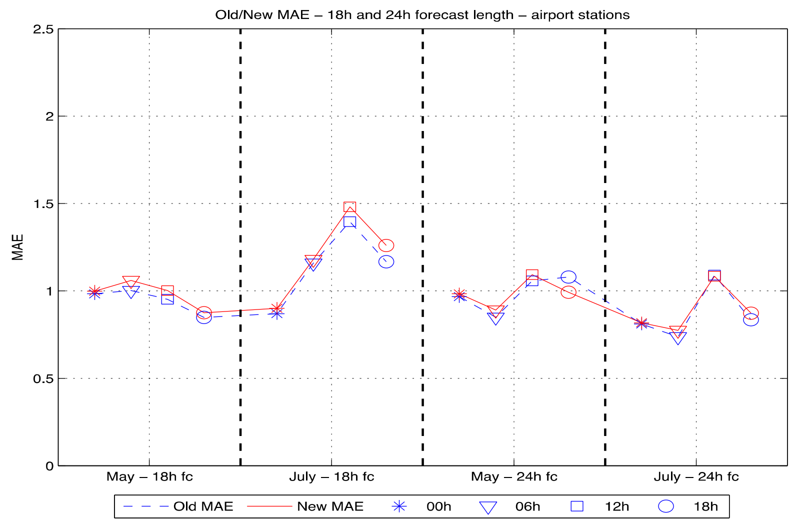


Old and New monthly MAE for each time of the day

06h and 12h forecast length – AS group



B2.2. For 18h and 24h forecast lengths

In blue: Percentage of MAE improvement *In red: percentage of Old MAE >1* *In Green: percentage of New MAE >1* *18h and 24h forecast length – AS group*	
Old and New monthly BIAS for each period of the day *18h and 24h forecast length – AS group*	
Old and New monthly MAE for each period of the day *18h and 24h forecast length – AS group*	

Appendix C: Number of observational records excluding missing values

		High Error Stations (HES)				Airport Stations (AS)			
		Forecast length, in hours				*Forecast length, in hours*			
Month	***Time of the day (in UTCs)***	***06***	***12***	***18***	***24***	***06***	***12***	***18***	***24***
MAY	***00h***	288	288	288	288	351	351	351	351
	06h	296	286	286	286	365	353	353	353
	12h	297	297	287	287	367	367	356	356
	18h	299	299	299	289	367	367	367	355
JUN	***00h***	260	260	260	260				
	06h	259	259	259	259				
	12h	255	255	255	255				
	18h	260	260	260	260				
JUL	***00h***	280	280	280	280	357	357	357	357
	06h	280	280	280	280	365	365	365	365
	12h	281	281	281	281	367	367	367	367
	18h	281	281	281	281	367	367	367	367
AUG	***00h***	233	233	233	233				
	06h	232	232	232	232				
	12h	233	233	233	233				
	18h	232	232	232	232				

Table C1: *Number of observational records per month, per forecast length and per time of the day.*

	High Error Stations (HES)				**Airport Stations (AS)**	
Month	**May**	**Jun**	**Jul**	**Aug**	**May**	**Jul**
00h	1153	1040	1120	932	1404	1428
06h	1154	1036	1120	928	1424	1460
12h	1168	1020	1124	932	1446	1468
18h	1186	1040	1124	928	1456	1468
Time of the day (UTCs)						

Table C2: *Number of observational records for all forecast lengths per month and per time of the day.*