Calculation of the Height of Stable Boundary Layers in Operational Models

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Abstract

Currently used and newly proposed calculation techniques for the height of the stable boundary layer (SBL), including the bulk-Richardson-number method, diagnostic equations for the equilibrium SBL height and a relaxation-type prognostic equation, are discussed from the point of view of their physical grounds and relevance to experimental data. Among diagnostic equations, the best fit to data exhibits an advanced Ekman-layer height model derived recently with due regard to the role of the free-flow stability. Its extension to non-steady regimes provides a prognostic equation recommended for use in operational models.

Keywords: boundary layer height, stable stratification, bulk Richardson number, air-pollution, weather prediction

1. Introduction

The height, $h$, of turbulent boundary layers (often called “mixing height”) is requested in a number of practical applications, first of all, in pollution-dispersion modelling, where the upper boundary of the turbulent layer could play a role of impenetrable lid for pollutants released at the surface. $h$ also appears as a mixing height scale in turbulence schemes within climate and weather prediction models.

Currently used $h$-calculation techniques are summarised in the final report of the working group 2 “Mixing Height Determination for Dispersion Modelling” of the EU COST Action 710 (Seibert et al., 1998, 2000). This document asserts essential uncertainties in specification of $h$ especially for stable boundary layers (SBLs) and general need for further research with emphasis on non-steady regimes and wave-turbulence interaction.

To some extent these features of the SBLs are already included in the SBL height formulation (Zilitinkevich et al., 2001a). Further analysis along this line is given in the present paper. The state of the art in the SBL height parameterisation can be found in the above quoted papers.
2. Critical Richardson number methods

2.1. Gradient Richardson number

As follows from the classical theory (Taylor, 1931) infinitesimal disturbances in a steady-state homogeneous stably stratified sheared flow decay if the gradient Richardson number $R_i$ exceeds a critical value $R_i_c$,

$$R_i = \frac{\beta (\partial \theta / \partial z)}{(\partial u / \partial z)^2 + (\partial v / \partial z)^2} > R_{i_c} = 0.25. \quad (1)$$

Here, $z$ is the height, $u$ and $v$ are the velocity components, $\theta_v = \theta + 0.61 T_0 q$ is the virtual potential temperature, $\theta$ is potential temperature, $q$ is specific humidity, $\beta = g / T_0$ is the buoyancy parameter, $g$ is the acceleration due to gravity, and $T_0$ is a reference value of the absolute temperature. The estimate $R_{i_c} = 0.25$ is derived from the perturbation analysis.

Strictly speaking, the above concept is not immediately applied to turbulent boundary layers, which are always heterogeneous in the vertical and often non-steady. Nevertheless a rather common practice is to employ the critical gradient Richardson number as a convenient tool for distinguishing between the planetary-boundary-layer interior, supposed to be essentially turbulent, and the free atmosphere, supposed to be non-turbulent or only weakly turbulent. Accordingly, the turbulent boundary layer height, $h_E$, is deduced from inequalities

$$R_i < R_{i_c} \text{ at } z < h_E \quad \text{and} \quad R_i > R_{i_c} \text{ at } z > h_E, \quad (2)$$

regardless the type of the boundary layer, stable or unstable.

Leaving aside the general applicability of the above method to heterogeneous flows, it obviously implies that the boundary layer is in the steady state. Hence the critical Richardson number method can provide, at best, the equilibrium height of the boundary layer, $h_E$, rather than its actual height, $h$.

In practical calculations the gradients on the r.h.s. of Eq. (1) are approximated by finite differences as $\Delta \theta / \Delta z$, $\Delta u / \Delta z$ and $\Delta v / \Delta z$, where the increments in $\theta$, $u$ and $v$ over the vertical distance $\Delta z$ are taken from measured or numerically simulated vertical profiles (e.g., Marion et al., 1991).

When the boundary-layer height is known from independent measurements or numerical simulations (e.g., using advanced turbulence closures), the Richardson number immediately above the boundary layer can be identified, and the method of estimating $h_E$ through Eqs (1) and (2) can be evaluated. A reasonable criterion of its robustness would be an empirical evidence that the “geophysical critical Richardson number” is not much variable.
Estimates of $R_i_c$ presented in Table 1 contradict this expectation: $R_i_c$ varies from 0.15 to 7.2. By this means the gradient Richardson number method in the above straightforward form is hardly justified.

### Table 1. Geophysical estimates of the critical gradient Richardson number, $R_i_c$.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$R_i_c$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor, 1931</td>
<td>0.25</td>
<td>Theoretical – for homogeneous flows</td>
</tr>
<tr>
<td>Webb, 1970</td>
<td>0.19 – 0.2</td>
<td>Deduced from conventional empirical constants in the Monin-Obukhov theory</td>
</tr>
<tr>
<td>Businger et al., 1971</td>
<td>0.21</td>
<td>Deduced from the Kansas experiment based constants in the Monin-Obukhov theory</td>
</tr>
<tr>
<td>Businger, 1973</td>
<td>0.15 – 0.5</td>
<td>From wind-tunnel and field data: turbulence is developed at $R_i&lt;0.15$ and decays at $R_i&gt;0.5$</td>
</tr>
<tr>
<td>Maryon and Best, 1992</td>
<td>1.3 (up to 7.2)</td>
<td>Using calculated $R_i(z)$ from numerical model NAME/UM and actual $h$ from radiosoundings</td>
</tr>
<tr>
<td>Straume et al. 1998</td>
<td>0.55</td>
<td>Best fit for the ETEX experiment data</td>
</tr>
</tbody>
</table>

### 2.2. Bulk and finite-difference Richardson numbers

An alternative, widely used method of estimating $h$ employs, instead of the gradient Richardson number $R_i$, Eq. (1), the boundary-layer bulk Richardson number, $R_{i_B}$, specified as

$$R_{i_B} \equiv \frac{\beta \Delta \theta \cdot h}{U^2}$$

through the wind velocity at the upper boundary of the layer, $U = \sqrt{u^2(h) + v^2(h)}$, and the virtual potential temperature increment across the layer, $\Delta \theta_v = \theta_v(h) - \theta_v(0)$. As common sense suggests, the SBLs can grow on the background of stable stratification only until $R_{i_B}$ achieves some critical value, $R_{i_{BC}}$. When this threshold is passed, the shear production of turbulent kinetic energy, characterised by the strength of wind $U$, becomes insufficient to overtake the energy losses, characterised by the buoyancy increment $\Delta \theta_v$. This reasoning immediately yields the formula (Mahrt, 1981; Troen and Mahrt, 1986)

$$h_E = \frac{R_{i_{BC}} U^2}{\beta \Delta \theta_v},$$

where $h_E$ is the equilibrium SBL height.

The fact that any version of the Richardson number method provides the equilibrium rather than actual SBL height deserves emphasising. Generally SBLs, especially over
urban or coastal areas, are non-steady. What follows from the above reasoning is only an indication that the actual SBL should have a tendency to evolve towards its equilibrium state with the height given by Eq. (4). It is obvious that the accuracy of the bulk Richardson number method can not be too high. Nevertheless, Eq. (4) gives reasonable order-of-magnitude estimates of $h$ taking $Ri_{bc}$ in the interval $0.2 < Ri_{bc} < 0.5$.

In view of rather uncertain specification of $Ri_{bc}$, numerous attempts were made to improve the method through a compromise between the gradient and the bulk Richardson number approaches. The idea of this development is to exclude the lower portion of the SBL and to determine a “finite-difference Richardson number”, $Ri_F$, on the basis of increments $\partial \theta_v = \theta_v(h) - \theta_v(z)$ and $\partial U = \sqrt{u^2(z) + v^2(z)}$ over the height intervals $z_i < z < h$ and $z_2 < z < h$. Clearly $Ri_F$ is nothing but a roughly estimated gradient Richardson number. Assuming the existence of its standard critical value, $Ri_{Fc}$, the equilibrium SBL height formulation becomes

$$h_E \approx \frac{(h_E - z_j)^2}{h_E - z_1} = \frac{Ri_{Fc} (\partial U)^2}{\beta \partial \theta_v}.$$  

(5)

There is no consensus in the choice of the lower reference heights. For example, $z_1 = 2$ m and $z_2 = 0$ in Holtslag et al. (1990), $z_1 = 30$ m and $z_2 = 0$ in Sørensen et al. (1996), $z_1 = z_2 = 20, 40$ or $80$ m in Vogelezang and Holtslag (1996). In numerical-weather-prediction (NWP) and other operational models, $z_1$ and $z_2$ are usually identified with the lower numerical-model level. Using any of these versions, the finite-difference critical Richardson number $Ri_{Fc}$ remains quite uncertain (see Table 2). Hence, to the authors’ understanding, the $Ri_{Fc}$–method can hardly be considered as an essential step forward compared to the $Ri_{bc}$–method.

Besides this principal difficulty, different authors employ different definitions of bulk or finite-difference Richardson numbers. As a result, inattentive readers could easily overlook what particular version of the Richardson number is implied – bulk, $Ri_{bc}$, or finite-difference, $Ri_{Fc}$, and moreover, what particular choice of $z_1$ and $z_2$ is implied in the definition of $Ri_{Fc}$. For reader’s convenience, a summary of the Richardson number based methods is briefly presented in Table 2.
Table 2. Alternative critical values of the bulk Richardson number $Ri_{Bc} \left( z_1 = z_2 = 0 \right)$ and the finite-difference Richardson number $Ri_{Fc}$.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$z_1$, m</th>
<th>$z_2$, m</th>
<th>$Ri_{(B,F)c}$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laikhtman, 1961</td>
<td>0</td>
<td>0</td>
<td>1.65</td>
<td>$Ri_B$ in terms of geostrophic wind; data from Main Geo. Obs. expeditions in Russia</td>
</tr>
<tr>
<td>Hanna, 1969</td>
<td>0</td>
<td>0</td>
<td>0.33-0.56</td>
<td>$Ri_B$ in terms of temperature gradients in lower 100 m; data from O’Neill, Nebraska</td>
</tr>
<tr>
<td>Melgarejo and Deardorff, 1974</td>
<td>0</td>
<td>0</td>
<td>Average 0.55 typical 0.3</td>
<td>Data from Wangara exp.; $h$ determined through the wind maximum height, $h_u$</td>
</tr>
<tr>
<td>Brost and Wyngaard, 1978</td>
<td>1</td>
<td>1</td>
<td>0.11-0.22</td>
<td>Data from measurements and 2nd order closure model</td>
</tr>
<tr>
<td>Anisimova et al., 1978</td>
<td>0</td>
<td>0</td>
<td>up to 7</td>
<td>Lab experiments with down slope drainage flows (analysed by Mahrt 1981)</td>
</tr>
<tr>
<td>Zeman, 1979</td>
<td>$\frac{1}{2}h$</td>
<td>$\frac{1}{2}h$</td>
<td>0.5</td>
<td>Data on nocturnal jets over the Great Plains, O’Neill; $h$ compared with the Brost – Wyngaard closure model</td>
</tr>
<tr>
<td>Mahrt et al., 1979</td>
<td>2</td>
<td>0</td>
<td>average 0.3-0.5 maximum 15</td>
<td>Data from Wangara, Risø, O’Neill and Haswell; $h$ compared with $h_u$</td>
</tr>
<tr>
<td>Mahrt, 1981</td>
<td>0</td>
<td>2</td>
<td>0.5 – 1.0</td>
<td>Typical values of $Ri_{Bc}$ or $Ri_{Fc}$ from different sources</td>
</tr>
<tr>
<td>Wentzel, 1983</td>
<td>2</td>
<td>0</td>
<td>0.33</td>
<td>Wangara data (mainly for radiation dominated SBLs) with different estimates of $h$</td>
</tr>
<tr>
<td>Troen and Mahrt, 1986</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>Data from LES (Deardorff model) and Wangara exp.</td>
</tr>
<tr>
<td>Byzova et al., 1989</td>
<td>0</td>
<td>0</td>
<td>0.6-1.0</td>
<td>Data on turbulence and mean profiles from 300-m tower, Obninsk, Russia, 1972-1974</td>
</tr>
<tr>
<td>Heineman and Rose, 1990</td>
<td>2</td>
<td>0</td>
<td>0.3-0.55 typical 0.33</td>
<td>Tethered balloon sounding, Filchner/Ronne Ice Shelf, Antarctica; $h$ compared with $h_u$, the Zilitinkevich (1972) SBL height scale, and the height, $h_u$, of the lowest $\theta$ gradient discontinuity</td>
</tr>
<tr>
<td>Authors and Year</td>
<td>Ri Range</td>
<td>h Range</td>
<td>Description</td>
<td></td>
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<tr>
<td>------------------</td>
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<td></td>
</tr>
<tr>
<td>Holtslag et al., 1990</td>
<td>0.25-0.5</td>
<td>Best fit for radiosounding data from de Bilt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Holtslag and Boville, 1993</td>
<td>0.5</td>
<td>Modelling and radiosonde data from several sites</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sørensen et al., 1996</td>
<td>0.14-0.24</td>
<td>Ri&lt;sub&gt;B&lt;/sub&gt; from either HIRLAM or radiosoundings, h from radiosoundings at weakly stable SBLs, Jægersborg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vogelezang and Holtslag, 1996</td>
<td>0.21–0.22, 0.30–0.32</td>
<td>(i) For nocturnal SBLs, (ii) for well-mixed SBLs – both from Cabauw-mast data and SODAR data (for h)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fay et al., 1997</td>
<td>0.38</td>
<td>Ri&lt;sub&gt;B&lt;/sub&gt; from German NWP model and actual h from either radiosoundings or 2&lt;sup&gt;nd&lt;/sup&gt; order closure model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Makshtas et al., 1998</td>
<td>0.4</td>
<td>Ri&lt;sub&gt;F&lt;/sub&gt; from aerological and balloon observations over Weddell sea; h compared with wind-maximum and inversion heights (h&lt;sub&gt;u&lt;/sub&gt; and h&lt;sub&gt;i&lt;/sub&gt;)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Andreas et al., 2000</td>
<td>0.4</td>
<td>Ri&lt;sub&gt;B&lt;/sub&gt; from radiosoundings at the Ice Station Weddell; h compared with h&lt;sub&gt;u&lt;/sub&gt; and h&lt;sub&gt;i&lt;/sub&gt;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Besides $\text{Ri}_B$ and $\text{Ri}_F$, some other versions of the Richardson number were considered. Thus Vogelezang and Holtslag (1996) proposed for near-neutral SBLs a version of $\text{Ri}_F$ modified by including in the velocity increment an additional term dependent on the friction velocity, $u^2_\ast$, namely:

$$\text{Ri}_F = \frac{\beta \delta \theta (h - z_1)}{((\delta U)^2 + b u^2_\ast)^{\frac{3}{2}}}.$$  \(\text{They took } b \approx 100 \text{ and } z_1 = z_2 = 20, 40 \text{ or } 80 \text{ m and recommended the critical values of } \text{Ri}_F \text{ equal to } 0.16-0.20 \text{ for nocturnal SBLs (type I) and } 0.25-0.28 \text{ for well-mixed SBLs (type II). Using data from the experiment TEBEX, Baltink and Holtslag (1997) considered one more version of this formulation with } z_1 = z_2 = 0.1 \text{ h and } \text{Ri}_{Fc} = 0.25.\)  

As clearly seen from the above discussion, any version of the Richardson number method displays the following weak points.

- Inherent uncertainty of the method: it provides an equilibrium rather than actual SBL height and inevitably becomes a poor approximation in non-steady regimes.
- Considerable uncertainty in the choice of appropriate critical Richardson numbers, which additionally degrades the accuracy of the method.

The first disadvantage can not be mastered within the Richardson number method. Instead, a prognostic SBL-height equation should be used to account for non-steady states (see Section 3).

The second disadvantage does not look hopeless. Wide spread in empirical estimates of the bulk or finite-difference Richardson numbers can naturally be treated as an indication that other parameters, besides the wind speed and the buoyancy increments, affect the equilibrium SBL height.

This idea is by no means new. Brutsaert (1972) has found that $\text{Ri}_{Bc}$ generally increases in the regimes with clear-air radiation cooling. Arya (1972) reported about pronounced difference in empirical estimates of $\text{Ri}_{Bc}$ for the shallow and the deep SBLs. Joffre (1981) disclosed a positive correlation between the critical bulk Richardson number, $\text{Ri}_{Bc} = \frac{\beta \Delta \theta h}{U^2}$, and the dimensionless combination $fh/\delta u$, where $\beta$ is the Coriolis parameter and $u_\ast$ is the friction velocity.

### 2.3. Earth’s rotation

Generally the role of the Earth rotation in the Ekman boundary layer is indisputable. However, the above correlation could at least partially be an artefact, as the SBL height $h$ appeared simultaneously on the $x$- and $y$-axis of Joffre’s diagrams.

Further investigation of the dependence of $\text{Ri}_{Bc}$ on $\beta$ is presented in Figure 1. Here, $\text{Ri}_{Bc}$ is plotted against two alternative dimensionless numbers, $\mu$ and $M$, characterising comparative roles of stratification and rotation,

$$\mu = \frac{u_\ast}{|\beta| L}, \quad \text{and} \quad M = \frac{\beta \Delta \theta_v}{|\beta| U}.$$  \(6\)


Figure 1. Empirical dependencies of the critical bulk Richardson number $\text{Ri}_{bc} = \beta \delta \theta h / U^2$ on alternative stratification/rotation parameters $\mu = u_c / |f|L$ and $M = \beta \Delta \theta_c / |f|U$, after Cabauw data: (a) $\text{Ri}_{bc}(\mu)$ and (b) $\text{Ri}_{bc}(M)$. 
Of these two, $M$ is composed of the bulk increments over the SBL (Section 2.2.1.2 in Zilitinkevich, 1970) and $\mu$, of the surface-layer parameters, namely, the friction velocity $u_*$, and Monin-Obukhov length scale,

$$L = \frac{-u_*^3}{B_i},$$

(7)

where $B_i = \beta F_{\theta v}$ and $F_{\theta v}$ are the near-surface turbulent fluxes of buoyancy and virtual potential temperature, respectively. Experimental data are taken from measurements performed in 1977-1979 on the 200-meter meteorological mast in Cabauw, the Netherlands (Niewstadt, 1984; Van Ulden and Wieringa, 1996; Vogeiezang and Holtslag, 1996). They include the mean vertical profiles measured at 8 levels between 2 and 200 meters) and the SODAR-measurement based SBL height, $h$.

Figure 1a does not show pronounced systematic dependence of $\text{Ri}_{bc}$ on $\mu$. This is not surprising bearing in mind that all data are taken from one and the same site, Cabauw, so that the Coriolis parameter is fixed and the representative range of $\mu$ is rather limited. A vague tendency of $\text{Ri}_{bc}$ to increase with increasing $\mu$ (indicated by dashed line) is not statistically ensured. Further analysis of this type of dependencies including data from measurements at different latitudes (together with data from lab experiments in rotating tanks and large-eddy simulations with variable $f$) would be useful.

A well-pronounced dependence of $\text{Ri}_{bc}$ on $M$ shown in Figure 1b does not say much about $\text{Ri}_{bc}$, but implicitly indicates that $h$ could depend on $f$. Indeed, $M = \text{Ri}_{bc} U/|f|h$, which is why $\text{Ri}_{bc}$ factually drops out from linear relationship $\text{Ri}_{bc} \propto M$. Thus taking $\text{Ri}_{bc} = 0.002 M$ (which is a reasonable approximation at $M > 50$) immediately yields $h \approx 0.002 U / |f| h$.

2.4. Static stability above the SBL

An external parameter, which is not included in the standard Richardson-number methods, is the Brunt-Väisälä frequency, $N$, in the free atmosphere immediately above the SBL,

$$N^2 = \beta \frac{\partial \theta}{\partial z} \text{ at } h < z < 2h.$$  

(8)

$N$ is evidently unimportant for the nocturnal SBLs. The latter develop on the background of well-mixed residual layers, which are near neutrally stratified. Then $N$ given by Eq. (8) is quite small. From the physical point of view, residual layers separate the nocturnal SBLs from the stably stratified free atmosphere. This prevents vertical propagation of internal waves and makes the SBL turbulence essentially local.

In contrast, long-lived SBLs typical of persisting stable stratification are in immediate contact with the free atmosphere. Then, if the free-flow stability is strong enough, the
SBL turbulence becomes essentially non-local due to wave-induced vertical fluxes of the kinetic energy and the squared buoyancy fluctuations. Zilitinkevich and Calanca (2000) and Zilitinkevich (2001) have shown that $N$ becomes an important governing parameter for this type of SBLs. It is only natural to expect that the critical Richardson numbers also depend on $N$.

For the surface-layer Richardson number this is already demonstrated by Zilitinkevich et al. (2001b). Moreover, a pronounced dependence on $N$ immediately follows from recent analysis of the finite-difference Richardson number $Ri_{Fc}$. Thus Vogelezang and Holtslag (1996) reported that $Ri_{Fc}$ for nocturnal SBLs lie in the interval 0.18 – 0.22 (depending the choice of values of $z_i$ and $z_2$), and for slightly stable well-mixed boundary layers, in the interval 0.23 – 0.32. This difference can be very naturally attributed to essentially different typical values of $N$ in the two cases: $N$ close to zero for shallow nocturnal SBLs (capped by residual layers) and $N$ of order $10^{-2}$ s$^{-1}$ for much deeper well-mixed SBLs (immediately bordering the free atmosphere).

Figure 2 based on the Cabauw data set (the same data as in Figure 1) presents empirical dependencies of the critical bulk Richardson number $Ri_{Bc}$ on the free-flow stability, employing two alternative dimensionless arguments,

$$P = \frac{N}{|f|} \quad \text{and} \quad Q = \frac{NU}{\beta \Delta \theta},$$

of which $P$ characterises comparative roles of the free-flow stratification and earth’s rotation and $Q$, comparative roles of the free flow and the SBL stratification. The Coriolis parameter is fixed, $f = 1.15 \cdot 10^{-4}$ s$^{-1}$, so that Figure 2a showing $Ri_{Bc}(P)$ presents an uncombed effect of $N$. In spite of the wide spread of data, both empirical plots $Ri_{Bc}(P)$ in Figure 2a and $Ri_{Bc}(Q)$ in Figure 2b suggest that the effect is significant. The linear- and the power-regression approximations, $Ri_{Bc} = 0.1371 + 0.0024P$ and $Ri_{Bc} = 0.0598M^{-0.7775}$ are shown by solid lines in Figures 2a and 2b, respectively. For practical use, the first one is recommended, namely,

$$Ri_{Bc} = 0.1371 + 0.0024 \frac{N}{|f|}. \quad (10)$$

The benefit of the use of $N$-dependent $Ri_{Bc}$ in the bulk Richardson number method is demonstrated in Figure 3. The correlation coefficient between the calculated and the observed SBL heights, $h_E$ and $h_{SBL}$, increases from 0.56 in the standard version of the method (with $Ri_{Bc} = 0.25$) to 0.62 in the improved version based on Eq. (10).
Figure 2. Empirical dependencies of the critical bulk Richardson number $Ri_{bc}$ on the dimensionless parameters $P = N / |f|$ and $Q = NU / \beta \Delta \theta_v$ involving the free-flow Brunt-Väisälä frequency $N$, after Cabauw data: (a) $Ri_{bc}(P)$ and (b) $Ri_{bc}(Q)$. 
Figure 3. Comparisons of the SBL heights, $h_{SBL}$ deduced from the Cabauw data and $h_E$ estimated through critical bulk Richardson numbers: (a) standard $Ri_{Bc} = 0.25$, the linear regression line is $h_E = 1.045h_{SBL}$; (b) $N$-dependent $Ri_{Bc} = 0.1371+0.0024N/|f|$, the linear regression line is $h_E = 0.987h_{SBL}$. 
2.5. Surface roughness

One more parameter evidently overlooked in the standard Richardson number methods is the surface roughness length, $z_{0u}$. Common sense guides us to suppose that the level of turbulent mixing in the SBLs with equal $U$ and $\Delta \theta_v$ should be higher over the surface with the higher roughness. Accordingly the SBL height should also be higher. In terms of the bulk Richardson number, $Ri_{bc}$, this reasoning suggests that $Ri_{bc}$ should increase with increasing $z_{0u}$. A traditional dimensionless argument involving $z_{0u}$ is the surface Rossby number $Ro_s = U / |f| z_{0u}$. The dependence of $Ri_{bc}$ on $Ro_s$ presented in Figure 4 does not contradict the above expectation (remember, all data are taken from Cabauw, so that differences in $Ro_s$ reflect differences in the wind speed rather than in $z_{0u}$). Anyhow, the linear-regression approximation of data points in Figure 4 reads

$$Ri_{bc} \approx 0.2586 - 10^{-7} \frac{U}{|f| z_{0u}}.$$  \hfill (11)

All things considered, the critical Richardson number methods can be recommended only for rough, order-of-magnitude estimates of the SBL height. More advanced SBL height formulations are discussed below.

3. Multi-limit SBL height equations

3.1 Diagnostic equations

Zilitinkevich and Mironov (1996) employed the turbulent kinetic energy (TKE) budget equation to derive a diagnostic multi-limit equation for the SBL height,

$$\left(\frac{f h_E}{C_n u_s}\right)^2 + \frac{h_E}{C_s L} + \frac{N h_E}{C_i u_s} + \frac{|f B_s|^{1/2} h_E}{C_{sr} u_s^2} + \frac{|N f|^{1/2} h_E}{C_{ir} u_s} = 1,$$  \hfill (12)

where $C_n$, $C_s$, $C_i$, $C_{sr}$ and $C_{ir}$ are dimensionless constants. A week point in the derivation was the use of a rather uncertain parameterisation of the energy dissipation. At the same time Eq. (12) looks quite attractive. It includes five known length scales. Three of them account for rotation, namely, $h_E \propto u_s / |f|$ for the neutral Ekman layer (Rossby & Montgomery, 1935), $h_E \propto u_s^2 / |f B_s|^{1/2}$ and $h_E \propto u_s / |f N|^{1/2}$ for the Ekman layers affected by the surface buoyancy flux (Zilitinkevich, 1972) and the free-flow stability (Pollard et al., 1973), respectively. Two more length scales, $L$ and $u_s / N$, appropriate for non-rotating stably stratified flows, were employed as the SBL depth scales by Kitaigorodskii (1960) and Kitaigorodskii and Joffre (1988). Zilitinkevich and Mironov (1996) obtained provisional estimations of the above dimensionless constants, $C_n \approx 0.5$, $C_s \approx 10$, $C_i \approx 20$, $C_{sr} \approx 1.0$, and $C_{ir} \approx 1.7$, using data from field measurements and large eddy simulations (LESs).
Figure 4. Empirical dependence of the critical bulk Richardson number $Ri_{bc}$ on the surface Rossby number, $Ro_s = U / |f| z_{0u}$, after Cabauw data. The linear regression line is $Ri_{bc} = 0.2586 - 10^{-7} (U / |f| z_{0u})$. 
In recent years, Eq. (12) became widely used in air pollution modelling (e.g., Robertson et al., 1996; Brandt, 1998; Linqvist, 1999; Baklanov, 1999; SUBMESO, METPRO and CTDPLUS models). This deserves further theoretical discussion and empirical verification.

In Figure 5a, the SBL heights, \( h_E \), calculated after Eq. (12) are compared with the actual SBL heights, \( h_{SBL} \), deduced from the Cabauw data set. The correlation can be improved by refinement of empirical constants but only slightly. Thus taking \( C_n = 0.3 \) instead of 0.5, Eq. (12) gives higher correlation coefficient and lower root mean square (RMS) error; however, the bias slightly increases and the SBL height remains underestimated (see Table 3 and Figure 5b).

**Table 3.** Empirical constant \( C_n \) in Eq. (12) after Cabauw data.

<table>
<thead>
<tr>
<th>( C_n )</th>
<th>Bias</th>
<th>RMS error</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-33.76</td>
<td>33.84</td>
<td>0.384</td>
</tr>
<tr>
<td>0.3</td>
<td>-35.91</td>
<td>32.05</td>
<td>0.387</td>
</tr>
</tbody>
</table>

More detailed empirical evaluation of Eq. (12) is given in Table 4 below.

From the theoretical point of view, the TKE budget underlying Eq. (12) was quite probably oversimplified through neglecting the effect of rotation on the energy dissipation. Moreover, straightforward analysis of the Ekman equations indicates that the scales \( L \) and \( u_*/N \), included as principal limits in Eq. (12), are not immediately applicable as the depth scales for turbulent boundary layers in rotating fluids.

More recently Zilitinkevich et al. (2001a) derived a refined multi-limit Ekman-layer height equation from the momentum equations given a modern formulation for the eddy viscosity (Zilitinkevich, 2001), which implicitly accounted for the TKE budget. The refined equation reads

\[
h_E = \frac{C_R u_*}{|f|} \left[ 1 + \frac{C_S^2 u_*^2 (1 + C_{uN} NL/ u_*)}{C_R^2 |f| L} \right]^{-1/2},
\]

where \( C_R, C_S, C_{uN} \) are the same type empirical constants as \( C_n, C_{sr}, C_{ir} \) in Eq. (12).

### 3.2. Vertical-motion correction and prognostic equation

Eq. (13) can be extended to account for the synoptic scale vertical motions through the large-scale vertical velocity, \( w_h \), at the SBL upper boundary. Thus, considering a relaxation equation for the actual SBL height (Zilitinkevich et al., 2001a),
Figure 5. Comparisons of the SBL heights, $h_{SBL}$ deduced from the Cabauw data and $h_E$ after Eq. (12): (a) with $C_n = 0.5$; (b) with $C_n = 0.3$. 

$h_e [m]$ 

$h_{SBL} [m]$ 

$h_{SBL} [m]$
\[
\frac{\partial h}{\partial t} + \mathbf{V} \cdot \nabla h - w_h = -C_E |f| (h - h_E),
\]
(14)

and keeping only the vertical advection term on the l.h.s. of this equation, a quasi-equilibrium SBL height, \( h_{QE} \), becomes

\[
h_{QE} = h_E + \frac{w_h}{C_E |f|},
\]
(15)

where \( C_E \) is one more empirical constant. With \( C_E \sim 1 \) (Zilitinkevich et al., 2001a), the correction term could be as large as \( \pm 100 \) m, which explains quite uncertain performance of any equilibrium SBL height formulations.

Eqs. (13) and (15) are not applicable to non-rotating Equatorial SBLs. When \( f \) tends to zero, Eq. (14) predicts unlimited growth of the SBL height. However, Equatorial SBLs never live very long. Then the SBL height becomes limited due to the finite periods of the SBL development on the background of the damping effect of capping inversions. Accordingly a climatological upper limit \( h \leq h_c \) should be put on the SBL height.

Deardorff (1972) was probably the first who employed this idea in a practically oriented boundary-layer height parameterisation. He specified \( h_c \) as the height of the tropopause (~15 km). As established by Larsen (2000), climatological upper limits for the boundary-layer height are always much less than 15 km and differ over different geographical sites (e.g., \( h_c = 3 \) km for Denmark).

Interpolation between reciprocals of \( h_{QE} \) and \( h_c \) provides a “corrected quasi-equilibrium SBL height” \( h_{CQE} \),

\[
\frac{1}{h_{CQE}} = \frac{1}{h_{QE}} + \frac{1}{h_c},
\]
(16)

where the second term on the r.h.s. is nearly always negligible except for the Equatorial region.

Figure 6 shows a scatter diagram for theoretical versus observed SBL heights (\( h_{CQE} \) versus \( h_{SBL} \)). Here, \( h_{SBL} \) is deduced from the Cabauw data; whereas \( h_{CQE} \) is calculated after Eqs. (13), (15), (16) with empirical constants \( C_R = 0.4, C_S = 0.75 \) and \( C_{uN} = 0.25 \) recommended by Zilitinkevich et al. (2001a). The constant \( C_E = 1 \) was not requested as the vertical advection term was not included in calculations. As Figures 5 and 6 suggest, \( h_{CQE} \) after Eqs. (13), (15), (16) is a better approximation than \( h_E \) after Eq. (12).

Qualitatively these two formulations are evaluated in Table 4 based on the Cabauw data. The table presents the bias (m), the RMS error (m) and the correlation coefficient for \( h_{CQE} \) after Eqs. (13), (15), (16), for \( h_E \) after Eq. (12) and for some other commonly
Figure 6. Comparisons of the SBL heights, $h_{SBL}$ deduced from the Cabauw data and $h_{CQE}$ after Eqs. (13), (15), (16) with $C_R = 0.4$, $C_s = 0.75$, $C_{aw} = 0.25$. 
used diagnostic SBL height equations. The basic scales given in this table are “RM” (Rossby and Montgomery, 1935) $u_*/|f|$ and “Z” (Zilitinkevich, 1972) $u_*^2 |fB_s|^{-1/2}$. Purely empirical formulas are not based on physical scales.

Table 4. Empirical evaluation of different SBL height equations

<table>
<thead>
<tr>
<th>Reference</th>
<th>Basic scale</th>
<th>SBL height equation</th>
<th>Bias</th>
<th>RMS error</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benkley &amp; Schulman, 1979</td>
<td>None</td>
<td>$h = 125 u_{10}$</td>
<td>208</td>
<td>264</td>
<td>0.48</td>
</tr>
<tr>
<td>Arya, 1981</td>
<td>Z</td>
<td>$h = 0.42 u_*^2</td>
<td>fB_s</td>
<td>^{-1/2} + 29.3$</td>
<td>64.0</td>
</tr>
<tr>
<td>Arya, 1981</td>
<td>RM</td>
<td>$h = 0.089 u_*/</td>
<td>f</td>
<td>+ 85.1$</td>
<td>103</td>
</tr>
<tr>
<td>Mahrt, 1982</td>
<td>RM</td>
<td>$h = 0.06 u_*/</td>
<td>f</td>
<td>$</td>
<td>-24.4</td>
</tr>
<tr>
<td>Niewstadt, 1984</td>
<td>None</td>
<td>$h = 28 u_{10}^{3/2}$</td>
<td>6.27</td>
<td>13.9</td>
<td>0.48</td>
</tr>
<tr>
<td>Niewstadt, 1984</td>
<td>Z</td>
<td>$h = 0.4 u_*^2</td>
<td>fB_s</td>
<td>^{-1/2}$</td>
<td>24.4</td>
</tr>
<tr>
<td>Zilitinkevich &amp; Mironov, 1996</td>
<td>Multi-scale</td>
<td>$h = h_E$ after Eq. (12)</td>
<td>-33.8</td>
<td>33.8</td>
<td>0.38</td>
</tr>
<tr>
<td>Zilitinkevich et al., 2001a</td>
<td>Multi-scale</td>
<td>$h = h_{CQE}$ after Eqs. (13), (15), (16) with $w_h = 0$</td>
<td>6.21</td>
<td>19.2</td>
<td>0.60</td>
</tr>
</tbody>
</table>

It is worth mentioning that $h_{CQE}$ - Eqs. (13), (15), (16) performs quite well in the cases with strong free-flow stability. Here, the bias and the RMS error become 3.8 m and 18.7 m, respectively.

It follows that $h_{CQE}$ after Eqs. (13), (15), (16) is a reasonable diagnostic SBL height formulation. It is recommended for use within 1-D models. Within 3-D models, the SBL height, $h$, can be calculated more accurately on the basis of Eq. (14). Employing $h_{CQE}$ instead of $h_E$ and accounting for the sub-grid scale horizontal motions through the horizontal diffusivity $K_h$, Eq. (14) becomes

$$\frac{\partial h}{\partial t} + \mathbf{V} \cdot \nabla h = -C_E |f| (h - h_{CQE}) + K_h \nabla^2 h,$$

where $\mathbf{V} = (u, v)$ is the horizontal velocity vector. The vertical advection term ($-w_h$) is included in $h_{CQE}$.

4. Concluding remarks

The SBL critical bulk Richardson number, $Ri_{bc}$, is not a constant. It evidently increases with increasing free flow stability and very probably depends on the surface roughness.
length and the Coriolis parameter. The Richardson-number-based calculation techniques can be recommended only for rough estimates of the SBL height.

For more accurate SBL height calculations within 1-D and 3-D models, respectively, the diagnostic formulation $h_{\text{CQE}}$ - Eqs. (13), (15), (16) and the prognostic formulation $h$ - Eq. (17) are recommended.

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