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# A mass and thermodynamic model for sea ice

Rasmus Tonboe

**Figure 1 Polarstern in Fram Strait spring 2003.**

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# Colophon

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Rasmus Tonboe

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# Abstract

A description of a thermodynamic model for snow on sea ice. The model is tested with meteorological data from the Arctic Ocean.

# Resumé

En beskrivelse af en termodynamisk model for sne på havis. Modellen er afprøvet med meteorologiske data fra det Arktiske Ocean.

# Introduction

The microstructure and mechanical strength of snow are related. Avalanche warning systems therefore use thermodynamic and micro-structural snow models with meteorological parameter input. Land snow cover thermodynamic models including microphysical parameters are described in e.g. Brun et al. (1989) (the CROCUS model) and Jordan (1991) (the SNTHERM model). Sea ice thermodynamic models such as Maykut & Untersteiner (1971) with the aim of simulating ice growth and the fluxes between the Ocean and the Atmosphere lack a detailed description of the microphysical parameters. In sea ice models, the snow and ice are taken as homogeneous slabs. The microphysical properties of snow and sea ice are of primary importance for microwave emission and scattering. This report describes a thermodynamic mass model for sea ice which is extended with a description of the microphysical parameters with particular significance for microwave remote sensing. The output from this model is used as input to microwave emission and scattering models computing brightness temperature and backscatter time series in terms of the meteorological input to the thermodynamic mass model. The aim is to study the annual variability of microwave signatures in terms of snow and ice properties. This report describes the thermodynamic model.

# General model description

The model computes the thermodynamic and mass state of the snow/ sea ice system using the meteorological input data. The functions used in the model are from the open literature selected and combined in a way that seems most realistic for sea ice. The initial snow and ice profile is ordered top to bottom in a text file with the following parameters for each layer: layer number, thermometric temperature [K], type [new snow, 1/ old snow, 2/ first-year ice, 3/ multiyear ice, 4], density [kg/$m^3$], thickness [m], scatterer correlation length [mm] (grain or inclusion size), salinity [ppt], liquid water content [$m^3/m^3$], surface roughness [mm], snow (0) or sea ice (1). The meteorological data input file has the following parameters: time [decimal days], surface air pressure [hPa], air temperature [K], wind speed [m/s], incoming short-wave radiation [W/$m^2$], incoming long-wave radiation [W/$m^2$], relative humidity [%], mass of precipitation [kg/$m^2$]. The programme is evoked by: energy_balance('snow/ice profile','met file')

Both the thermal conductivity and specific heat of snow and ice are functions of temperature. To solve this problem it is necessary to use finite difference. Between the meteorological data updates the model is correcting the snow and ice properties at a 600 seconds (1/6 hours) time-step. It has not been possible to simulate the development of all snow and ice properties relevant for microwave remote sensing such as snow and ice surface roughness. Thermodynamic processes related to surface roughness are simply not well quantified and perhaps related to kinematic processes not described in the model.

It has been necessary to develop certain special features in order to handle the physics of the snow and sea ice system:
1) Snow on sea ice may contain brine which is accounted for in the computation of the thermal properties.
2) Ice growth or melt (from beneath) progresses as a function of the energy balance of the sea ice bottom layer. Salinity is a function of growth rate.
3) brine pocket size which are primary scatterers in first-year ice are determined by temperature.

# Snow and sea ice thermal properties

The thermal properties include the thermal conductivity and the specific heat. The thermal conductivity of snow is a function of its density and temperature and sea ice its salinity and temperature. The specific heat of snow/ slush/ saline ice is a function of the fractions of ice and brine and its temperature if there is any brine or liquid water.

## Snow and saline ice thermal conductivity

The thermal snow conductivity, $k_s$, is computed using Makshtas (1998) eq. 14, i.e.

$$k_s = 2.845\times10^{-6}\rho^2 + 2.7\times10^{-4}2^{\frac{T_s-233}{5}},$$

where $\rho$ is the snow density [kg/m$^3$] and $T_s$ is the snow temperature [K].

The thermal conductivity of saline ice, $k_i$, is computed using Makshtas (1998) eq. 12, i.e.

$$k_i = 2.03 + \frac{0.12\,S}{T_i - 273.0},$$

where S [ppt] is the salinity and $T_i$ the ice temperature [K].

## The specific heat of snow, saline slush, or saline ice

The specific heat [J/kgK] of saline snow or ice mixture, c, is a simplification of eq. 5.4, p. 73 in Doronin & Kheisin (1977), i.e.

$$c = c_{pureice}\frac{M_{ice}}{M} + c_{brine}\frac{M_{brine}}{M} + L_w M_{brine}\frac{\partial V_{brine}}{\partial T},$$

where $c_{pureice}$ is the specific heat of pure ice (2113 J/kgK), $c_{brine}$ is the specific heat of brine (4217 J/kgK), $M_i$ is the mass of pure ice, M is the total mass, $M_{brine}$ is the mass of brine, $L_w$ is the latent heat of fusion (0.334x10$^6$ J/kg) and $dV_{brine}/dT$ is gradient of brine volume change around a given temperature.

# Radiation balance

In the Maykut & Untersteiner (1971) "standard case" the net short and long wave radiation balance is about neutral at the end of the year. The net long wave radiation balance is negative during summer and winter while short wave energy input is significant (positive) only during summer.

## Short wave energy

Short wave energy penetrate the snow surface and is absorbed in the snow and ice profile. The albedo $\alpha$ is a function of snow density, $\rho_{surface}$ (Lefebre et al., 2003) i.e.

$$\alpha = 0.58 - 4.35\times10^{-4}(\rho_{surface} - 920).$$

The first-year and multiyear ice ice albedo is 0.75. The snow extinction coefficient, $y_e$, is set according to snow density,$\rho_s$, and snow grain size diameter, $x_{gs}$, (Jordan et al., 1999) i.e.

$$\gamma_e = 3.795 \times 10^{-3} \frac{\rho_s}{\sqrt{x_{gs}}} .$$

The first-year ice extinction coefficient is 2.0 and the multiyear ice extinction coefficient is 2.5. The deposit of shortwave energy in each layer of thickness z, is the difference between energy at the top ($Q_a$) and at the bottom ($Q_b$) of the layer, i.e.

$$Q_b = Q_a e^{\frac{-\gamma_e}{z}} .$$

## Longwave energy

The longwave energy balance at the surface is the difference between emitted and absorbed incoming radiation, i.e.

$$Q_{lw} = e_s Q_{il} - e_s k_b T_s^4 ,$$

where $e_s$ is the surface emissivity (0.98), $Q_{il}$ the incoming longwave radiation, $k_b$ is Stephan Boltzmans constant ($5.67051 \times 10^{-8} W/m^3K^4$), and $T_s$ is the surface temperature [K].

# Atmospheric / surface and conductive heat fluxes

The latent, sensible and precipitation heat fluxes exchange heat between the surface and the atmosphere. The conductive heat flux exchange heat within the snow and ice profile.

## Latent heat

Latent heat is the energy added to the surface by condensation of water vapour or removed by evaporation. The magnitude of latent heat loss is greater during summer than winter (Maykut & Untersteiner, 1971). The formulation for latent heat, $Q_{lh}$, by Garratt (1992) is used, i.e.

$$Q_{lh} = 0.622 \rho_{air} L C_e u \frac{rh \; es_a - es_0}{P} ,$$

where

$$es_a = 6.112 e^{17.67 \frac{T_a - 273.15}{T_a - 29.65}} ,$$

and

$$es_0 = 6.112 e^{17.67 \frac{T_0 - 273.15}{T_0 - 29.65}} ,$$

$T_a$ is the air temperature [K], $T_0$ is the surface temperature, $\rho_a$ is the density of air (1.25 $kg/m^3$) (Garratt, 1992; p. 284), L is the latent heat of vaporisation ($2.502 \times 10^6$ J/kg)(Garratt, 1992; p. 284), $C_e$ is the bulk transfer coefficient ($0.55 \times 10^{-3}$)(Maykut, 1986; p. 415), es with subscript a or 0 is the partial water vapour pressure in the air and at the surface respectively, rh is the relative humidity at measurement height, P is the surface air pressure [Pa].

## Sensible heat

Sensible heat, $Q_{sensible}$, is the energy added or removed from the surface by momentum fluxes in the lowest atmosphere. The net $Q_{sensible}$ is negative during summer and positive during winter (Maykut & Untersteiner, 1971):

$$Q_{sensible} = \rho_a c_{air} C_s u \, (T_a - T_0) \, ,$$

where $\rho_{air}$ is the density of air (1.25 kg/m$^3$) (Garratt, 1992; p. 284), $c_{air}$ is the specific heat of air (1005.0 J/kgK) (Garratt, 1992; p. 284), $C_s$ is the momentum transfer coefficient (1.2x10$^{-3}$) (Maykut, 1986; p. 415), u is the wind speed [m/s] and T is the temperature [K].

## Conductive heat flux

The conductive heat flux is simply the thermal conductivity, k, times the temperature gradient, i.e.

$$Q_{conductive} = k \, \frac{\partial T}{\partial z} \, ,$$

## Precipitation heat

Precipitation in the form of rain add heat to the upper layer proportional to the rain mass and temperature difference between the air ($T_a$>0°C) synonymous with the rain water temperature and the surface layer ($T_0$<=0ºC) (Brun et al., 1989), i.e.

$$Q_{rain} = M_{rain} c_{water} (T_a - T_0) \, ,$$

where $M_{rain}$ is the mass of rain [kg/m$^2$] and $c_{water}$ the specific heat of water (4217.0 J/kgK).

# Liquid water flow

The flux of liquid water in the snow, U, is a function of snow permeability, $\kappa$, viscosity of water (0ºC) $\mu_{water}$ (1.79x10$^{-3}$ N/sm$^2$), the liquid water content, $S_w$ and the irreducible water content, $S_{wi}$ which is set to 4% (Jordan et al. 1999) i.e.

$$U = \rho_{water} g \frac{\kappa}{\mu_{water}} \left( \frac{S_w - S_{wi}}{1 - S_{wi}} \right)^3 \, ,$$

where the permeability is a function of grain size $x_{gs}$ and density of the snow, $\rho_{snow}$., i.e.

$$\kappa = 0.077 \, x_{gs}^2 \, e^{-0.0078 \rho_{snow}} \, .$$

# Snow grain size

The relatively thin snow cover in the Arctic Ocean can give large temperature gradients within the snow. A strong temperature gradient and the resulting vapour flux forms depth hoar at the ice/snow interface or near the snow surface (Garraty, 1992). Snow metamorphosis is particularly rapid when the snow is melting. The net result of snow melt, whether the snow is liquid saturated (>14%) or just moist i.e. pendular, is a snow grain size increase. The grain size growth processes in saturated snow is characterised by mass transfer or cannibalizing, i.e. the larger snow grains grow at the expense of small grains. The growth process in moist snow is a combination of grain clustering, vapour diffusion and grain surface diffusion (Colbeck, 1982). Barber et al. (1995) indicate that the growth process in the upper mid portion of the snow pack is due to water vapour diffusion while the lower snow pack just above the ice surface is wet and snow grains grow by grain clustering.

### Snow grain metamorphosis in dry snow

The formation of depth hoar in snow is a function of time t, temperature T, temperature gradient dT/dz and density $\rho$. The mean grain size, x, is a function of these parameters (Marbouty, 1980) i.e.

$$x(t) = x_0 + f(T)\, g\left(\frac{\partial T}{\partial z}\right) h(\rho)\, \psi(t) .$$

The functions f, g, h attain values between 0 and 1. The time dependent $\Psi(t)=0.09$ [mm/day]. Marbouty (1980) further noticed that the density is constant during depth hoar formation.

## Snow grain size metamorphosis in wet snow

The snow grain growth in moist and wet snow is a function of time, t, and liquid water content,f (Brun, 1989). The singular snow grain volume, i.e.

$$v(t) = v_0 + v' t \quad ,$$

where $v_0$ is the initial grain volume. The factor v' is a function of the liquid water content, f, i.e.

$$v' = v'_0 + v'_1 f^3 ,$$

where $v'_0 = 1.28 \times 10^{-8}$ $mm^3/s$ and $v'_1 = 4.22 \times 10^{-10}$ $mm^3/s$.

## Scatterer size in first-year ice

The primary scatters in first-year ice are brine pockets. Their size are related to the volume of brine (Light et al., 2003). The brine pocket size is therefore set proportional to the brine volume.

## Ice growth from the bottom

The ice thickness is increasing during winter in response to the negative energy budget (thermodynamic growth). The growth rate is proportional to the energy budget of the bottom layer. Its salinity [ppt], S, is a function of growth rate, dz/dt, (Nakawo & Sinha, 1981), i.e.

$$S = 32.0 \frac{0.12}{0.12 + 0.88\, e^{-4.2\times 10^4 \frac{\partial z}{\partial t}}}$$

## Snow compaction

The compaction is computed using (a corrected version of) Brun et al. (1989). The compaction rate is a function of snow load $F_{snow}$ [N/m$^2$], density $\rho$ [kg/m$^3$] and temperature $T_s$ [K], i.e.

$$z = \left(1 - F_{snow}\frac{t}{a}\right) z,$$

where

$$a = 10.0 \times 10^4 e^{23.0\rho - 0.1(T_s - T_{melt})}.$$

# Snow density

## New snow density

The density [kg/m$^3$], rho, of added snow layers is a function of air temperature [K], $T_{air}$, and wind speed [m/s], u, (Jordan et al., 1999; eq. 14), i.e.

if $T_{air}$>260.15K,

$$\rho = 500\left(1.0 - 0.951 e^{-1.4\left(278.15 - T_{air}\right)^{-1.15}} - 0.008u^{1.7}\right),$$

else,

$$\rho = 500\left(1.0 - 0.904 e^{-0.008u^{1.7}}\right).$$

## Snow density due to metamorphosis

The snow density, $\rho$, due to metamorphosis is computed using Sturm & Holmgren (1998) eq. 3, i.e.

$$\rho = \rho + \frac{F_{snow}\,\rho\, t}{\mu_o e^{k\rho}},$$

where k=0.02 is a constant, $\mu_0$=8.5x10$^6$ is the snow viscosity [Ns/m$^2$].

# Model results

The model is tested with ECMWF 6 hourly reanalysis meteorological data in a point between Fram Strait and the North Pole (87.5N;0.0E) September 2000 – June 2001.

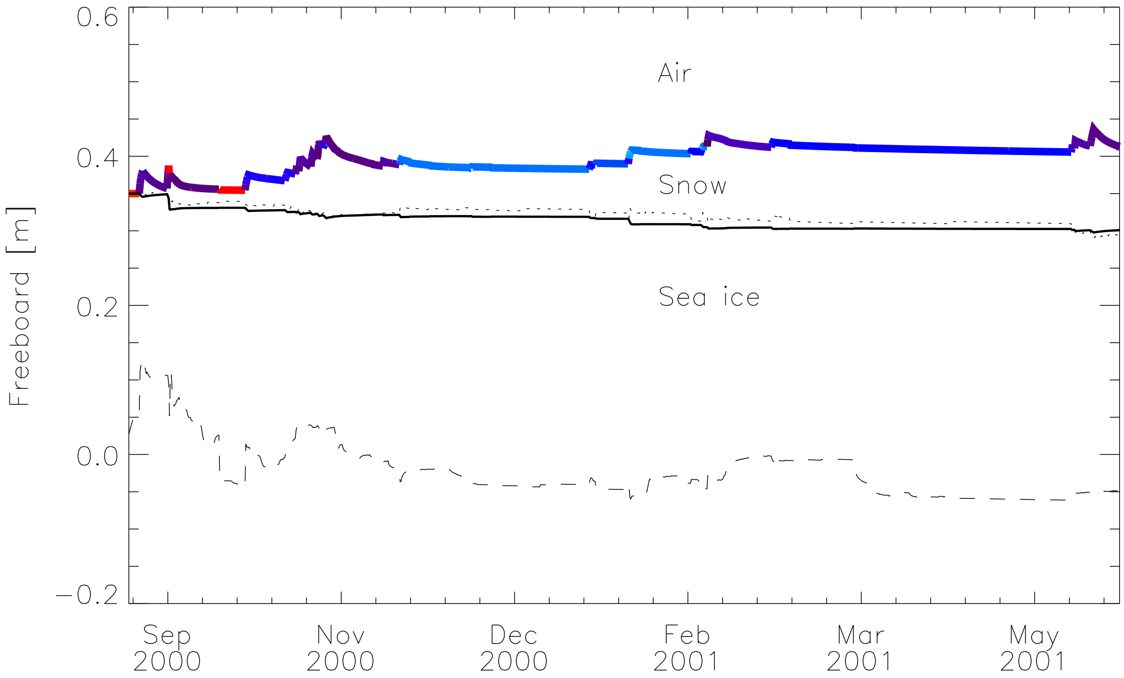


Figure 2. The seasonal snow cover on a 3.5m thick multiyear ice floe. The ice floe thickness is kept constant but its buoyancy is computed using the density profile. The snow surface density the dark colours indicate low density (100-250 kg/m$^3$), blue medium (250-350 kg/m$^3$) and red ice surface. The snow and ice profile is used as input to microwave backscatter model and the Ku-band penetration depth is shown with the dashed line and the altimeter effective scattering surface is shown with the dotted line.

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