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A new scheme for parameterization of turbulent surface fluxes in the stably stratified atmospheric boundary layer

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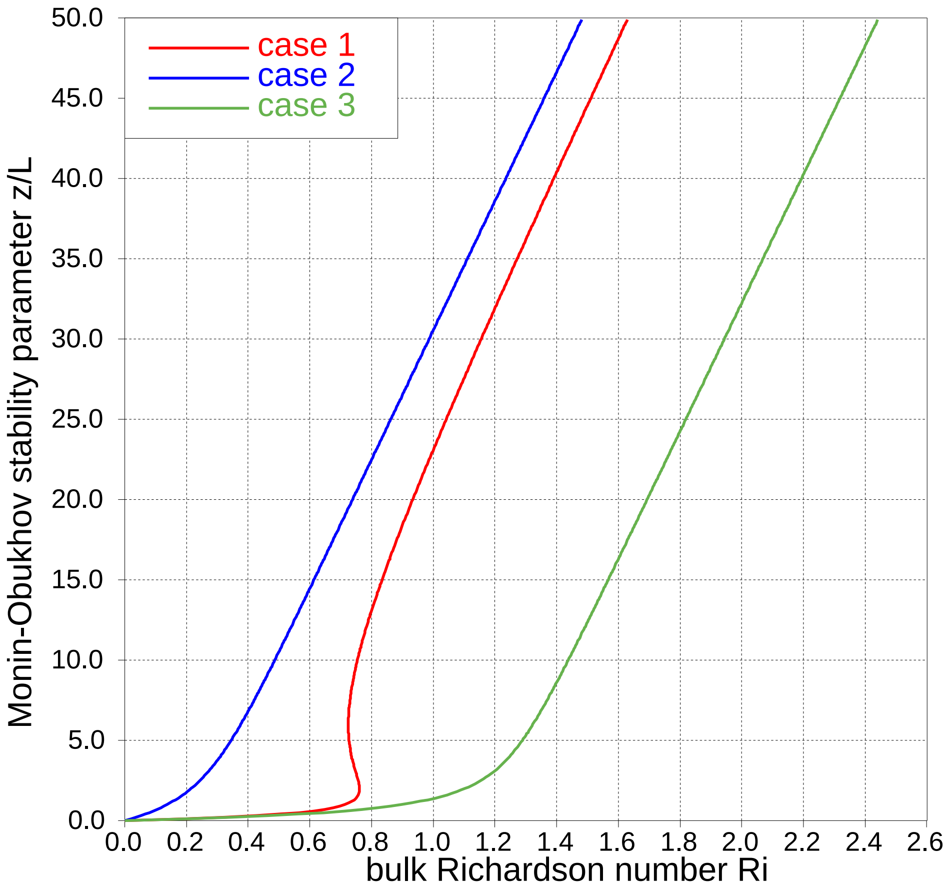

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Dansk Resume

I nærværende artikel foreslås for det stabilt lagdelte grænselag en parameterisering af den turbulente overflade flux af impuls, sensibel og latent varme, som direkte beregner disse størrelser ud fra et bulk Richardson tal (Ri). Det nye er, at det ikke først er nødvendigt at beregne udvekslingskoefficienter, som funktion af Ri eller Monin-Obukhov stabilitetsparameteren. Metoden tager højde for både det svagt stabile og det stærkt stabile grænselag og er ligeledes konsistent med total turbulent energi lukning og indeholder ikke et kritisk Ri. Metoden kræver løsning af en 3. grads ligning, som beskriver sammenhængen mellem Ri og Monin-Obukhov stabilitetsparameteren. Hvis ruhedslængden for temperatur benyttes i definitionen på Ri skal ruhedslængden for temperatur være større en en vis brøkdel af ruhedslængden for vind for at undgå multiple løsninger. Hvis derimod overfladetempraturen beregnes i ruhedshøjden for vind i udtrykket for Ri er der for $\mathrm{Ri} > 0$ en og kun en positiv løsning løsning til 3. grads ligningen.

Abstract

In the present article a parameterization of turbulent surface fluxes based on the bulk Richarson numer (Ri) is proposed for the stably stratified, nocturnal planetary boundary layer (PBL). The parameterization works both for the strong and weak mixing regimes in stable stratification and does not involve any critical bulk Ri. It is a novel feature of the method that calculation of drag and exchange coefficients as functions of either Ri or the Monin-Obukhov stability parameter is unnescesary. Furthermore, the method is consistent with total turbulent energy closure and requires solution of a cubic equation, describing the relationship between Ri and Monin-Obukhov stability parameter. If the roughness length for temperature is applied in the definition of Ri non-unique solutions to the kubic equation may exist, and to obtain a unique solution it is necessary to constrain the temperature roughness to be larger than a certain fraction of the roughness length for momentum. This constraint disappears if the bottom temperature in Ri instead is calculated at the roughness height for momentum.

1. Introduction

Present day numerical weather prediction models (NWP) still have so coarse horizontal and vertical resolution that they are unable to resolve the majority of turbulent motions. The latter are important for mixing of momentum and scalars in the atmosphere, not least in its boundary layer, which is the layer influenced by surface friction adjacent to the surface of the Earth. The turbulent fluxes of momentum, sensible and latent heat at the interface between the atmosphere and ocean/ice/land surface (henceforth referred to as turbulent surface fluxes) are of particular importance since they both play a significant role in the surface budget equations and serve as lower boundary fluxes in the NWP turbulence schemes. In the latter schemes the effect of turbulence on the resolved state of the atmosphere and its underlying surface is parameterized by applying a mixture of empirical and theoretically based equations involving both turbulence and mean state variables (i.e. variables resolved by the model). In the present article a parameterization of turbulent surface fluxes based on the bulk Richarson numer is proposed for the stably stratified, nocturnal planetary boundary layer (PBL). The parameterization works both for the strong and weak mixing regimes in stable stratification (Zilitinkevich et al., 2007b; Mauritzen and Svensson, 2007) and does not involve any critical bulk Richardson number. It requires solution of a cubic equation, describing the relationship between the bulk Richardson number and the Monin-Obukhov stability parameter.

2. Theoretical background

The Monin-Obukhov stability parameter ($\zeta = \mathrm{z}/\mathrm{L}$), derived from similarity arguments (e.g. Arya, 2001) applied for a steady state, horizontally homogeneous surface layer (bottom part of thePBL), is a key parameter in the parameterization of turbulent surface fluxes in NWP models. In the parameter ζ, z is the height above the surface and

$$L = -\frac{k_\theta}{k}\frac{(\tau_0/\rho_0)^{3/2}}{\beta H_{s0}/(c_p\rho_0)} \tag{1}$$

is the Obukhov length scale. Note that in many applications $\mathrm{L_0 = L/k}$ is used. In (1) $\tau_0 = \rho_0 \mathrm{u}_*^2$ and $\mathrm{H_{s0}} = -\mathrm{c_p}\rho\theta_*\mathrm{u}_*$ are the turbulent surface fluxes of momentum and sensible heat, respectively. Further, $\beta = \mathrm{g}/\overline{\theta}_0$ is the buoyancy parameter, $\mathrm{k} \approx 0.4$ the Von Karman constant, $\mathrm{k}_\theta \approx \mathrm{k}$ a corresponding constant for temperature, $\overline{\theta}_0$ the surface potential temperature, ρ_0 surface air density and $\mathrm{c_p}$ the specific heat capacity of dry air at constant pressure. Finally, u_* and θ_* are typical turbulent velocity and temperature fluctuations at the surface. The former is usually called the surface friction velocity, and in the classical similarity approach both u_* and θ_* are treated as constants in the surface layer. In NWP models drag and heat transfer relations are used to calculate the turbulent surface fluxes. In general the latter relations depend on both mean state and turbulence variables. The turbulence variables appear in the drag and heat transfer laws as stability functions depending on ζ. The equations become implicit in ζ, and the latter parameter therefore must be calculated by iteration. Computational savings can be obtained if the drag and heat transfer relations are made explicit by applying stability functions that only depends on mean state variables. This is done in for example the HIRLAM (High Resolution Limited Area Model) model, where the stability functions are dependent of a bulk Richardson number defined by

$$Ri = \beta\frac{\Delta\overline{\theta} z_r}{\vec{V}_r^2}, \tag{2}$$

where $\mathrm{z_r}$ is some reference height above the surface (in NWP models often the height of the lowest model level), $\Delta\overline{\theta} = \overline{\theta}(\mathrm{z_r}) - \overline{\theta}_0$, $\overline{\theta}_0 = \overline{\theta}(\mathrm{z}_{0\theta})$, $\mathrm{z}_{0\theta}$ the roughness length for temperature and $\vec{\mathrm{V}}_\mathrm{r}$ the horizontal wind velocity at height $\mathrm{z = z_r}$.

Note that a dry (in the sense without water vapor) boundary layer is considered here. The method described in the next section can be generalized to include the effect of water vapor below saturation. This is done in section 6.

3. The method

Equation (2) can be rewritten as

$$Ri = \zeta \frac{x_\theta - \psi_\theta(\zeta)}{(x_0 - \psi_u(\zeta))^2}, \tag{3}$$

where $\mathrm{x_0} = \ln \frac{\mathrm{z_r}}{\mathrm{z_0}}$, $\mathrm{x}_\theta = \ln \frac{\mathrm{z_r}}{\mathrm{z}_{0\theta}}$ and $\mathrm{z_0}$ and $\mathrm{z}_{0\theta}$ are the roughness lengths for momentum and temperature, respectively. The functions

$$\psi_u = \int_{\zeta_0}^{\zeta} (1 - \phi_u)\, d \ln \zeta \tag{4}$$

and

$$\psi_\theta = \int_{\zeta_{0t}}^{\zeta} (1 - \phi_\theta)\, d \ln \zeta \tag{5}$$

are obtained by vertical integration (in terms of ζ) of the non-dimensional profile functions

$$\phi_u(\zeta) = \frac{kz}{u_*} \frac{\partial |\vec{V}|}{\partial z} \tag{6}$$

and

$$\phi_\theta(\zeta) = \frac{k_\theta z}{\theta_*} \frac{\partial \overline{\theta}}{\partial z} \tag{7}$$

for a steady state, horizontally homogeneous surface layer. The universal dependence of the non-dimensional profile functions on ζ follows from the similarity hypothesis that the mean state vertical gradients of wind and potential temperature only depend on the buoyancy parameter, the height above the surface and kinematic surface fluxes of momentum and sensible heat. This hypothesis appears to be reasonable for the surface layer of the stably stratified nocturnal PBL. If generalized similarity, which includes the long-lived stable PBL, is applied, the surface fluxes are replaced by height dependent local fluxes and both the impact of the Coriolis parameter and the free-flow static stability must be considered. This makes the non-dimensional profile functions in (6) and (7) more complex (Zilitinkevich and Esau, 2007c).

Based on results from field experiments, e.g. Wangara (Clarke et al., 1971) and Kansas and Minnesota (Kaimal and Wyngaard, 1990) the following approximations have been widely used in the stably stratified surface layer:

$$\phi_u = 1 + a_m \zeta \tag{8}$$

$$\phi_\theta = 1 + a_h \zeta, \tag{9}$$

where $\mathrm{a_m}$ and $\mathrm{a_h}$ are constant coefficients (e.g. Arya, 2001). Equation (3) then becomes

$$Ri = \zeta \frac{x_\theta + a_h(\zeta - \zeta_{0\theta})}{(x_0 + a_m(\zeta - \zeta_0))^2} \approx \zeta \frac{x_\theta + a_h \zeta}{(x_0 + a_m \zeta)^2} \tag{10}$$

Equation (10) has the property that ζ approaches infinity as Ri approaches the critical number $\mathrm{Ri_c} = \mathrm{a_h}/\mathrm{a_m^2}$. Often $\mathrm{a_m} = \mathrm{a_h} = \mathrm{k} \cdot 5.0$ have been used in (10), limiting the range of Ri to $\mathrm{Ri} < \mathrm{Ri_c} = 0.2$. However, in a surface layer with weak turbulence rather strong vertical gradients of $\overline{\theta}$ can be maintained at low wind speeds such that $\mathrm{Ri} > \mathrm{Ri_c}$. In practice the parameterization of turbulent surface fluxes is done such that the stability functions in the drag and heat exchange relations decrease asymptotically to zero as Ri or ζ go towards infinity. It is a weakness of this approach that the proposed stability functions are rather ad hoc. Is there a better fit to observational data in the stably stratified surface layer than that given by (8) and (9)? Several researchers have addressed this question. Zilitinkevich and Esau, 2007a have proposed to replace $\mathrm{a_h}$ in (9) with $\mathrm{a_h}(\zeta) = \mathrm{a_{h1}} + 2\mathrm{a_{h2}}\zeta$. This was done using a generalized stability parameter. The modification changes (10) to

$$Ri = \zeta\frac{x_\theta + a_{h1}\zeta + a_{h2}\zeta^2}{(x_0 + a_m\zeta)^2}. \tag{11}$$

Rearrangement leads to

$$\zeta^3 + A\zeta^2 + B\zeta + C = 0, \tag{12}$$

where $\mathrm{A} = (\mathrm{a_{h1}} - \mathrm{a_m^2 Ri})/\mathrm{a_{h2}}$, $\mathrm{B} = (\mathrm{x_\theta} - 2\mathrm{a_m x_0 Ri})/\mathrm{a_{h2}}$ and $\mathrm{C} = -\mathrm{x_0^2 Ri}/\mathrm{a_{h2}}$ are real coefficients depending on Ri and/or $\mathrm{x_\theta}$ and $\mathrm{x_0}$.

4. Solution of the cubic equation

The character of the tree roots of (12) for any given combination of Ri, $\mathrm{x_0}$ and $\mathrm{x_\theta}$ depends on the discriminant $\mathrm{D} = \mathrm{Q}^3 + \mathrm{P}^2$, where

$$Q = \frac{3B - A}{9} \tag{13}$$

and

$$P = \frac{9AB - 27C - 2A^3}{54}. \tag{14}$$

If $\mathrm{D} > 0$, (12) has one real root and two conjugate complex roots. If $\mathrm{D} < 0$ there are three unequal real roots and if $\mathrm{D} = 0$ at least two of these roots are equal. Since $\zeta > 0$ in stable stratification, valid real roots must be positive. The tree roots ζ_1, ζ_2 and ζ_3 in (12) satisfy the equations

$$\zeta_1 + \zeta_2 + \zeta_3 = -A, \tag{15}$$

$$\zeta_1\zeta_2 + \zeta_2\zeta_3 + \zeta_3\zeta_1 = B, \tag{16}$$

$$\zeta_1\zeta_2\zeta_3 = -C. \tag{17}$$

Table 1 shows how the roots in (12) depend on the signs of the coefficients A, B, C and the discriminant D, as deduced from (15), (16) and (17). Note that C is negative, whereas A, B and D may be either positive or non-positive.

Table 1: $+/-\mathrm{nr}(\mathrm{n} = 1, 2, 3)$ means n positive/negative real roots r and $2\mathrm{c}$ means two conjugate complex roots. + and - for the coefficients A,B,C and the discriminant D means positive and non-positive values for these quantities, respectively.

CASE	A	B	C	D	ROOTS
1	+	+	-	+	+1r, 2c
2	+	+	-	-	+1r, -2r
3	+	-	-	+	+1r, 2c
4	+	-	-	-	+1r, -2r
5	-	+	-	+	+1r, 2c
6	-	+	-	-	+3r or +1r,-2r
7	-	-	-	+	+1r, 2c
8	-	-	-	-	+1r, -2r

According to Table 1 there is only one real positive solution for ζ, except in case 6. In the latter case there may be multiple (either 1 or 3) positive real solution/s for ζ, which means that a unique relation between ζ and Ri may possibly not exist for certain values of Ri, $\mathrm{x_0}$ and $\mathrm{x_\theta}$. This is illustrated in Figure 1, where case 1 shows 3 positive solution within a rather narrow Ri-interval. A parameterization based on Ri is unable to handle cases with multiple solutions for ζ. Exclusion of case 6 in Table 1 is a sufficient (but not necessary) condition for only one positive root ζ, i.e. for a unique relation between Ri and ζ. Coefficient A changes sign at $\mathrm{Ri_a} = \mathrm{a_{h1}}/\mathrm{a_m}^2$ and coefficient B changes sign at $\mathrm{Ri_b} = \chi/2\mathrm{a_m}$, where $\chi = \mathrm{x_\theta}/\mathrm{x_0}$. The most simple sufficient condition is therefore to let $\mathrm{x_\theta}$ be constrained by the condition $\mathrm{Ri_a} \geq \mathrm{Ri_b}$ or $\chi \leq 2\mathrm{a_{h1}}/\mathrm{a_m}$, which eliminates cases 5 and 6 in Table 1. It can be noted that formal restrictions on χ disappears if $\mathrm{a_{h1}} = 2\chi/\mathrm{a_m}$, but theoretical support for taking this step is lacking. Zilitinkevich and Esau, 2007a, suggested $\mathrm{a_{h1}} = 1.6$, $\mathrm{a_{h2}} = 0.1$ and $\mathrm{a_m} = 2.0$, yielding $\chi \leq 1.6$. The latter condition implies $\mathrm{z_{0\theta}} \geq \mathrm{z_0}(\mathrm{z_0}/\mathrm{z_r})^{0.6}$. The value $\mathrm{a_m} = 2.0$ is consistent with a maximum flux Richardson number of ≈ 0.2 in extremely strong stable stratification. The latter has been determined from experimental large eddy simulations (LES) and direct numerical simulations (DNS) (Zilitinkevich et al., 2010).

It can further be noted that a non-unique solution of the kubic equation disappears if $\mathrm{z_{0\theta}}$ in the definition of Ri is replaced by $\mathrm{z_0}$, since the unique solution requirement then is $\mathrm{a_{h1}} \geq 1$. In fact it appears wrong to apply 3 levels ($\mathrm{z_{0\theta}}$, $\mathrm{z_0}$ and $\mathrm{z_r}$) in the definition of Ri.

5. The parameterization

The proposed parameterization (with $\mathrm{x_\theta}$ retained in (11)) is done in three steps. In the first step ζ is calculated from Ri by solving the third order equation (12) with the constraint $\mathrm{z_{0\theta}} \geq \mathrm{z_0}(\mathrm{z_0}/\mathrm{z_r})^{0.6}$. As discussed above the coefficient $\mathrm{a_{h1}} = 1.6$ might alternatively be replaced by $\mathrm{a_{h1}} = 2\chi/\mathrm{a_m}$. Then χ is free to be determined from physical arguments, but this step implies that $\mathrm{a_{h1}}$ no longer is a universal constant. Figure 1 shows an example of the impact on the solution for ζ by the latter change. In the shown case the effect is marginal for $\mathrm{Ri} < 0.6$, but becomes substantial for $\mathrm{Ri} > 0.7$. In the second step the kinematic surface momentum flux $\tau_0/\rho_0 = \mathrm{u_*^2}$ is obtained from

$$|\vec{V}_r| = \frac{u_*}{k}(x_0 + a_m\zeta). \tag{18}$$

The x- and y-components of the the momentum flux become

$$u_{*x}^2 = \left(u_*\frac{\overline{u}_r}{|\vec{V}_r|}\right)^2, \tag{19}$$

$$u_{*y}^2 = \left(u_*\frac{\overline{v}_r}{|\vec{V}_r|}\right)^2. \tag{20}$$

The latter relations are valid, since turning of the mean wind with height can be neglected in the surface layer, where the Coriolis force is much smaller than the frictional force and the horizontal pressure gradient force. Finally, in the third step the kinematic surface sensible heat flux $(\mathrm{Hs_0}/\rho\mathrm{c_p})$ is calculated from (1). The main advantage of the proposed parameterization is that there is no need for any surface drag and heat exchange relations involving more or less ad hoc stability functions that overcomes the critical Richardson number problem encountered by applying (8) and (9). The accuracy of the proposed parameterization depends on the accuracy to which the non-dimensional profile functions in (6) and (7) are determined and to the extent the similarity hypothesis is valid. The modifications of the non-dimensional profile functions in (6) and (7) suggested by Zilitinkevich and Esau, 2007a appear to be a step forward in accuracy, since results - as those shown

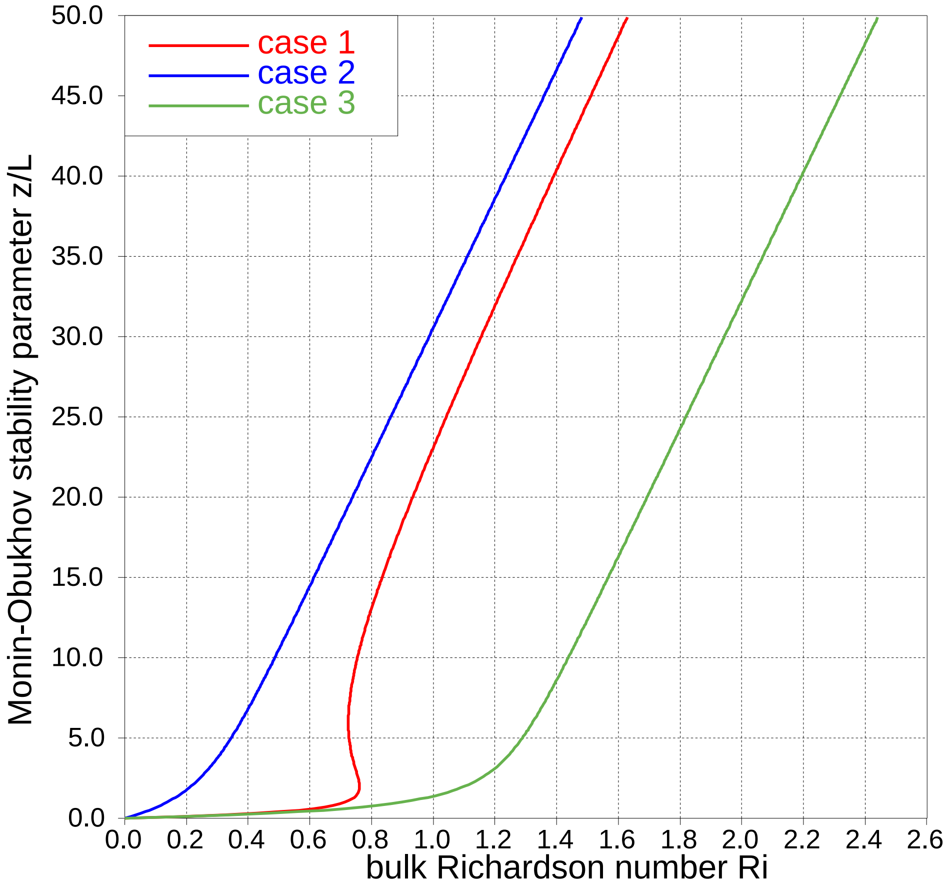

Figure 1: Case 1 (red) has a non-unique relationship between the bulk Richardson number and the Monin-Obukhow stability parameter. Three positive solutions exists for $\zeta = \mathrm{z}/\mathrm{L}$ in the range from $\mathrm{Ri} \approx 0.73$ to $\mathrm{Ri} \approx 0.77$. The case is rather extreme with $\mathrm{z}_0 = 3\,\mathrm{m}$, $\mathrm{z}_{0\theta} = 0.3\,\mathrm{mm}$ and $\mathrm{z}_\mathrm{r} = 30\,\mathrm{m}$. Case 2 (blue) is a 'normal' case ($\mathrm{z}_0 = 0.03\,\mathrm{m}$, $\mathrm{z}_{0\theta} = 3\,\mathrm{mm}$) and Case 3 (green) is as Case 1, except replacement of $\mathrm{a}_{\mathrm{h}1} = 1.6$ with $\mathrm{a}_{\mathrm{h}1} = 2\chi/\mathrm{a}_\mathrm{m}$. In Case 1 and 3, $\chi = 4$ and in Case 2, $\chi = 4/3$.

in Figure 1 - at least qualitatively agrees with total turbulent energy closure for the stably stratified PBL (Zilitinkevich et al., 2007c). The non-physically based constraint on $\mathrm{z}_{0\theta}$ that prevents multiple solutions and thus guarantees a unique relation between ζ and Ri might be a disadvantage. However this constraint only appears to be of concern over surfaces with large roughness bluff elements, as typical for urban areas. In this respect replacement of $\mathrm{a}_{\mathrm{h}1} = 1.6$ with $\mathrm{a}_{\mathrm{h}1} = 2\chi/\mathrm{a}_\mathrm{m}$ appears attractive over a surface dominated by large bluff elements since any formal (non-physical) constraint on $\mathrm{z}_{0\theta}$ disappears.

6. Generalization valid for the stably stratified unsaturated PBL

Moisture is taken into account by replacing θ with the virtual potential temperature

$$\theta_v = \theta(1 + \epsilon q), \tag{21}$$

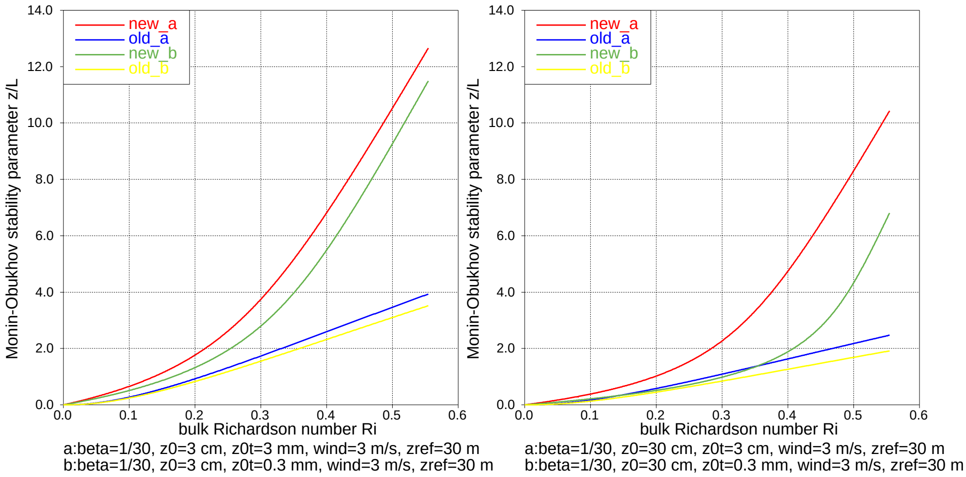

Figure 2: Monin-Obukhov stability parameter as function of the bulk Richardson number for the proposed parameterization (new) and the parameterization applied in the operational DMI-HIRLAM as briefly described in the Appendix (old). The wind speed is $3\,\mathrm{m\,s^{-1}}$ at reference height $\mathrm{z_r} = 30\,\mathrm{m}$ and $\beta = 1/30$. Left: (a) $\mathrm{z_0} = 0.03\,\mathrm{m}$ and $\mathrm{z_{0\theta}} = 0.003\,\mathrm{m}$, $\chi = 4/3$ and (b) $\mathrm{z_{0\theta}} = 0.0003\,\mathrm{m}$, $\chi = 5/3$; Right: (a) $\mathrm{z_0} = 0.3\,\mathrm{m}$ and $\mathrm{z_{0\theta}} = 0.03\,\mathrm{m}$, $\chi = 3/2$ and (b) $\mathrm{z_{0\theta}} = 0.0003\,\mathrm{m}$, $\chi = 2.5$.

where q is specific humidity of moist air, $\epsilon = (\mathrm{m_d}/\mathrm{m_w} - 1) = 0.61$ and $\mathrm{m_d}$ and $\mathrm{m_w}$ are the mean molecular masses of dry air and water vapor, respectively. The Obukhov length now takes the form

$$L_v = -\frac{k_b}{k}\frac{(\tau_0/\rho_0)^{3/2}}{F_{b0}}, \tag{22}$$

and the virtual bulk Richardson number is

$$Ri_v = \beta\frac{\Delta\overline{\theta}_v z_r}{\vec{V}_r^2} = \frac{\Delta\overline{b} z_r}{\vec{V}_r^2}, \tag{23}$$

where $\Delta\overline{\mathrm{b}} = \overline{\mathrm{b}}(\mathrm{z_r}) - \overline{\mathrm{b}}_0$ and $\overline{\mathrm{b}} = \beta\overline{\theta}_\mathrm{v}$. The buoyancy flux $F_{b0} = \beta F_{\theta 0} + g\epsilon F_{q0}$, contains the kinematic sensible heat flux at the surface $F_{\theta 0} = \beta H_{s0}/(c_p\rho_0)$ and the surface moisture flux $F_{q0} = H_{l0}/(\rho_0 L_e)$. $\mathrm{H_{s0}} = \rho_0\mathrm{c_p}\overline{\theta'\mathrm{w}'}_0$ is the surface sensible heat flux and $\mathrm{H_{l0}} = \rho_0\mathrm{L_e}\overline{\mathrm{q}'\mathrm{w}'}_0$ is the surface latent heat flux. In the latter $\mathrm{L_e}$ is latent heat of vaporization.

The similarity hypothesis for the surface layer is now that the mean state vertical gradients of wind, virtual potential temperature, potential temperature and specific humidity depend only on the hight above the surface, the buoyancy parameter and the surface fluxes of buoyancy and kinematic momentum.

Accordingly, the profile functions in (6) and (7) are replaced by

$$\phi_u(\zeta_v) = \frac{kz}{u_*}\frac{\partial|\vec{V}|}{\partial z} \tag{24}$$

and

$$\phi_b(\zeta_v) = \frac{k_b z}{b_*}\frac{\partial\overline{b}}{\partial z} = \frac{k_b z}{\theta_{v*}}\frac{\partial\overline{\theta}_v}{\partial z}, \tag{25}$$

respectively. In (24) and (25) $\zeta_v = z_r/L_v$ is a modified Monin-Obukhov stability parameter that takes into account the influence of moisture and in (25) $\mathrm{b_*} = \beta\theta_{\mathrm{v*}}$ and $\mathrm{k_b} = \phi_\mathrm{b}(\zeta_\mathrm{v} = 0)$ corresponds to k_θ in (7). Note that use of k_b in (25) is valid if $k_\theta = k_q$, where the latter plays the same role in the non-dimensional moisture profile as k_θ in (7). As for the dry case $\psi_u(\zeta_v) = 1 + a_m\zeta_v$ and $\psi_{\theta v}(\zeta_v) = 1 + a_{h1}\zeta_v + a_{h2}{\zeta_v}^2$. Noting that $F_{b0} = b_* u_*$, the virtual bulk Richardson number can then be written

$$Ri_v = \zeta_v \frac{x_b + a_{h1}\zeta_v + a_{h2}{\zeta_v}^2}{(x_0 + a_m\zeta_v)^2}, \tag{26}$$

which has the same form as (11) if it is further assumed that $z_{0b} = z_{0\theta} = z_{0q}$, the latter implying $x_b = x_\theta = x_q$. These assumptions have been widely used in NWP turbulence parameterizations. For a given Ri_v the solution for ζ_v is therefore the same as for ζ in the dry case with $Ri = Ri_v$. In shifting from the dry to the moist case two additional steps are involved in the proposed parameterization. Both ζ_v and the kinematic surface momentum flux is obtained in the same way as for the dry case. In the third step the kinematic surface buoyancy flux is obtained from

$$F_{b0} = -\frac{k_b}{k}\frac{{u_*}^3}{z_r}\zeta_v \tag{27}$$

Since $F_{b0} = F_{\theta 0}\,(\beta + \epsilon g F_{q0}/F_{\theta 0}) = F_{\theta 0}\left(\beta + \epsilon g(\overline{q}_r - \overline{q}_0)/(\overline{\theta}_r - \overline{\theta}_0)\right)$ the kinematic surface sensible heat flux is calculated from

$$F_{\theta 0} = \frac{F_{b0}}{\beta + g\epsilon(\overline{q}_r - \overline{q}_0)/(\overline{\theta}_r - \overline{\theta}_0)} \tag{28}$$

in step four. Finally, in step five the surface moisture flux is calculated from

$$F_{q0} = F_{\theta 0}\frac{\overline{q}_r - \overline{q}_0}{\overline{\theta}_r - \overline{\theta}_0}. \tag{29}$$

In (28) and (29) $\overline{\theta}_0$ and $\overline{q}_0$ are evaluated at the roughness heights for temperature and moisture, respectively. If they instead are evaluated at the momentum roughness height, which means that x_θ is replaced by x_0 in (11), the sensible heat flux at the surface is obtained by solving the cubic equation for the dry ($\theta_v = \theta$) Ri. Next the surface buoyancy flux is obtained by solving the cubic equation for the moist Ri, and finally the surface moisture flux can be calculated from the buoyancy and sensible heat flux.

7. Conclusions

A new method for calculation of turbulent surface fluxes of momentum, sensible heat and moisture has been developed for the stably stratified PBL. The method is based on calculation of the Monin-Obukhov stability parameter (z/L) from the bulk Richardson number (Ri). The method requires solution of a cubic equation. In order to avoid multiple solutions it is necessary to constrain the temperature (and moisture) roughness in such a way that they in case of a constant $a_{h1} = 1.6$ must be larger than a certain fraction of the momentum roughness. This constraint disappears if the constant a_{h1} is replaced by $a_{h1} = 2\chi/a_m$ or if the temperature and specific humidity both are calculated at the momentom roughness height in the bulk Richardson number for the lowest model layer. One advantage of the proposed method is that it makes use of recently adjusted non-dimensional profile functions for wind and temperature valid for the horizontally homogeneous surface layer. The latter functions have been obtained as best fits to observational data as well as data obtained from Large Eddy Simulation (LES) and they have been shown to be at least qualitatively consistent with total turbulent energy closure for the stable PBL. A second advantage is that an intermediate step involving drag and exchange coefficients is avoided. The latter coefficients are usually specified as functions of either z/L or Ri, but to the author's knowledge consistency with the adjusted non-dimensional profile functions has so far not been considered. In fact, it is shown that the new method and the more traditional method applied in DMI-HIRLAM (briefly described in the

Appendix) gives significantly different relations between z/L and Ri, particularly at large Ri, characterizing the weak mixing regime in stable stratification. Further, the proposed method appears to be computationally more efficient. This becomes an issue if stability dependent displacement height and momentum roughness are introduced in the surface flux parameterization, since the latter step implies an iterative solution for the surface fluxes.

Appendix

In the operational NWP model at DMI (DMI-HIRLAM, Sass et al.,2002)the kinematic surface fluxes are calculated by using

$$F_{\gamma 0} = \overline{w'\gamma'}_0 = C_\gamma \Delta\overline{\gamma}|\overrightarrow{V_r}|, \tag{30}$$

where $\gamma = u, v, \theta, q$ and C_γ are drag/exchange coefficients specified as

$$C_\gamma = C_{mn}\left(1 + \ln\frac{z_0}{z_{0t}} / \ln\frac{z_r}{z_0}\right) S_\gamma. \tag{31}$$

S_γ are stability functions depending on the bulk Richardson number. The stability function for the velocity components (momentum) is

$$S_u = S_v = \left(1 + \frac{a_m Ri}{(1 + b_m Ri)^{1/2}}\right)^{-1}, \tag{32}$$

and for sensible heat and moisture

$$S_\theta = S_q = \left(1 + a_\theta Ri(1 + b_\theta Ri)^{1/2}\right)^{-1}. \tag{33}$$

$C_{mn} = (k/\ln z_r/z_0)^2$ is the neutral drag coefficient and the the constants have the values $a_m = a_\theta = 10.0$ and $b_m = b_\theta = 1.0$.

1

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